VOLUME XLIII NUMBER 4

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

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MAY 1916

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ON AN APPARATUS AND METHOD FOR THERMO-ELECTRIC MEASUREMENTS IN PHOTOGRAPHIC PHOTOMETRY. I

By HARLAN TRUE STETSON

The desirability of purely physical methods for the determination of the magnitudes of stars has long been felt among astronomical workers in the field of photometry. Although the introduction of polarizing and wedge photometers has greatly increased the precision of visual work, systematic errors of considerable proportions may nevertheless escape detection where in the end the eye is the sole interpreter of stellar brightness. Aside from the unavoidable differences in spectral sensitiveness of individual eyes, the errors introduced in consequence of the well-known "Purkinje effect" involve considerable uncertainty in the photometric observations over wide ranges of magnitudes.

From the very introduction of the dry plate into astronomy it was seen that the size of the stellar image upon the plate might be taken as an index of the star's magnitude. As early as 1857 Bond[1] made use of the parabolic formula

$$Pt+Q=y^2$$

[1] *Astronomische Nachrichten*, **49**, 81, 1857.

to demonstrate the relation between exposure time t and the diameter y of the photographic image, P and Q being constants of the plate used. Later investigations of Charlier[1] showed close agreement between magnitudes and measured diameters when the relation was logarithmically expressed:

$$m = a - b \log D.$$

Investigations at Greenwich and elsewhere have seemed to indicate that a square-root relation was applicable to a wider range of conditions as regards plates and instruments than the logarithmic expression would satisfy, and accordingly the well-known form has found wide acceptance:

$$m = a - b\sqrt{D}.$$

Given the relation above, the precision of results depends upon the accuracy with which D may be measured. Since the images at best show no well-defined periphery, the principal source of error in measuring is the uncertainty of locating the extremities of the diameter to be measured. The difficulty is augmented if the images are elongated or poorly defined. Again, the same eye may pass different judgments upon large and small images in the same field. The amount of agreement obtained by different observers using the same method is commendable and indicates the degree of reliability of the results.

Other methods for the reduction of magnitudes include the scale method of estimation employed extensively at the Harvard College Observatory,[2] and the method of extra-focal images.[3] In the latter case, the plates being taken at a considerable distance from the focus, the star-disks are appreciably all of the same size and the difference of magnitude is determined by comparison of the opacity of the images with a calibrated photographic wedge, by means of a Hartmann micro-photometer or other similar device.

The encouraging results obtained in the use of the selenium cell for the direct measurements at the telescope as developed by

[1] *Publikationen der Astronomischen Gesellschaft*, No. 19.

[2] *Harvard Annals*, **18**, 120; **71**, 4.

[3] *Astrophysical Journal*, **26**, 244, 1907.

Minchin,[1] and by Stebbins,[2] and the still more recent investigations with the photo-electric cell, particularly those of Elster and Geitel,[3] Guthnick,[4] and others, indicate a far-reaching advance in the sensitiveness of photometric methods. At present, however, such methods are confined chiefly to stars of the first few magnitudes and lose advantages otherwise gained by photography.

The possibility of the application of some physical method for the measurements of photographic magnitudes from the rapidly accumulating collection of astronomical negatives has often been suggested, and emphasized particularly on various occasions by E. C. Pickering[5] in connection with the photometric researches of the Harvard Observatory. Accordingly, a series of experiments was begun by the writer in 1911 at the Wilder Laboratory of Dartmouth College, in search of suitable apparatus for the problem in hand. It is with the results obtained from this series of experiments and their application to the determination of stellar magnitudes that the present paper has to deal.

DESCRIPTION OF APPARATUS

The general principle involved is to measure the energy absorbed from a beam of light by the silver grains in the stellar image on a photographic plate, and to interpret such absorption in terms of stellar magnitude. The thermopile, bolometer, radio-micrometer, and radiometer as "detectors" were respectively considered. After some preliminary experiments and deliberation the thermopile was selected for the work as the instrument best combining simplicity and sensitiveness with convenience of manipulation.

It was seen from the outset that the apparatus in question would be more serviceable if adapted to the measurement of original negatives, rather than made dependent upon positive copies. Accordingly the scheme adopted was to restrict the region of the negative in the immediate vicinity of the star to be measured by a very small circular diaphragm, allowing the unobstructed

[1] *Proc. Roy. Soc.*, **58**, 142, 1895.

[2] *Astrophysical Journal*, **32**, 185, 1910.

[3] *Physikalische Zeitschrift*, **12**, 609, 1911; **14**, 1287, 1913.

[4] *Astronomische Nachrichten*, **196**, 357, 1913.

[5] *Harvard Annals*, **71**, 5.

light-beam to register its energy in a corresponding manner. When a successful combination of light-source, diaphragm, projecting lenses, and thermopile had been obtained, the whole apparatus was assembled, and placed in a convenient portable form. The diagram (Fig. 1) will serve to make clear the arrangement, and is reproduced as a suitable design for a more permanent and elegant form of the apparatus herein described.

Light from a source L passes through the condenser C and is brought to a focus on the surface of a metallic plate S, forming the stage of the instrument, upon which the photographic plate is laid. In the center of the stage is placed a small pinhole-diaphragm d, which serves the purpose of isolating a small portion of the plate in the immediate neighborhood of the stellar image to be measured. A projection lens P of short focal length is then used to project an image of d, and the stellar image with which it is in contact, upon the surface of a thermopile T connected to a galvanometer. Stellar images of different sizes corresponding to known magnitudes are placed in turn upon the diaphragm and the corresponding change in the galvanometer deflection is noted. This affords means for the calibration of the instrument. A detailed description of the method of reduction will be presented shortly. As considerable difficulty was encountered before the instrument could be relied upon for consistent readings with the degree of accuracy sought, the essential features in the design of the apparatus in its final form will be mentioned.

In regard to the thermo-element, it was seen at once that a surface-element would be necessary, of small heat-capacity, hence short period, and of the maximum sensitiveness. After futile efforts to obtain a suitable element in the market, the requirements were met by a thermopile specially designed and constructed for the purpose by W. W. Coblentz, of the Bureau of Standards, Washington. The element is a bismuth-silver combination of symmetrical design, having a circular receiving surface 5 mm in diameter, and a resistance of 2.3 ohms.[1] Tests reveal a sensitiveness

[1] For a detailed description of the construction of bismuth-silver thermo-couples, see "Instruments and Methods Used in Radiometry," by W. W. Coblentz, *Bulletin of the Bureau of Standards*, **9**, 15, 1912.

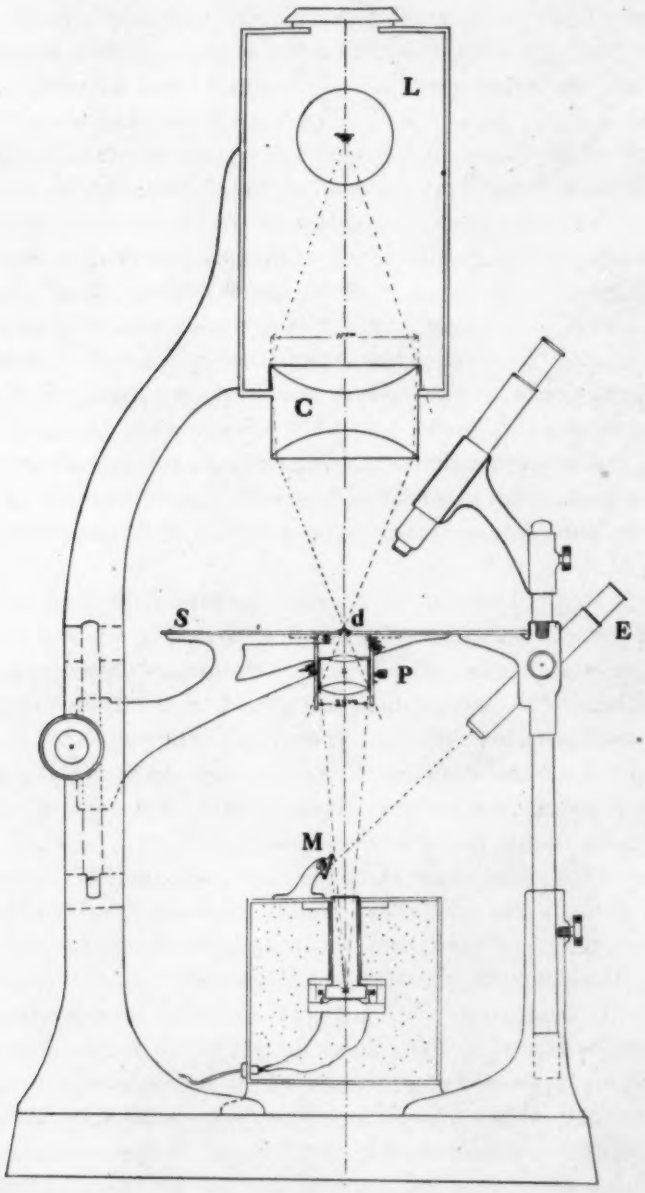


FIG. 1

of one microwatt per 0.18 microvolt, and a period of but a few seconds' duration. The element was incased in a brass receptacle and the whole mounted in the interior of a box of nonconducting material packed with cotton wool, the opening to the well through which the radiation passes being closed with a thin glass plate. Such insulation has proved wholly satisfactory under all ordinary conditions of temperature. With the seclusion of the thermopile, it became necessary to introduce a small mirror at *M* to reflect light from the element into a telescope of short focus at *E*, which serves to locate the stellar image upon the receiver.

Any error due to imperfect centering of the stellar image upon the thermopile is avoided by using the finding eyepiece and making several settings for each star. The agreement is generally better among the image-readings than among the plate-readings, as the latter are distributed more or less uniformly about the image in question, and represent the transparency for slightly different parts of the film.

As at best but a small part of the light from the source is transmitted through the restricting diaphragm *d* (Fig. 1), it is seen that the light-source used must combine intensity with smallness of dimension. The arc, though complying beautifully with these requirements, becomes entirely unsuited for the purpose, on account of its marked unsteadiness. Since it was hoped for the sake of compactness and convenience to use a direct-deflection method, an incandescent lamp operated from a storage-battery of 50 volts was sought. The most convenient luminant tested thus far has been found to be a 50 c.-p. stereopticon-bulb with a carbon filament spirally wound. Such a source when projected may be adjusted to give a satisfactory illumination over the diaphragm, uniformity of light being obtained by so orienting the bulb that projected images of the neighboring coils of the filament overlap slightly. The Nernst lamp at first appeared most promising, but in the final arrangement was abandoned in favor of the type described. Interviews with lamp-makers in consideration of filaments of special design have given little promise of improvement. The ideal source would be an incandescent lamp with a disk or spherical source some two or three millimeters in diameter, of about the intrinsic brightness

of the carbon lamp. The 50-volt stereopticon-bulb has the advantages of a filament of large radiating surface, and ease of duplication, and when operated from storage-batteries gives very satisfactory results.

In general any change in voltage is kept under control by connecting a precision voltmeter across the terminals of the lamp. An adjustable rheostat is provided for regulating the voltage when necessary. With storage-batteries of sufficient capacity, little error is introduced from this source, and any progressive or gradual fluctuations in voltage are duly corrected for in the method of reduction, as the plate-readings take into account both variation in the transparency of the film and any gradual change in the intensity of the source. As the plate-readings for each star are necessary in any instance, and form a convenient check on the readings, further precautions beyond the employment of storage-batteries and a sensitive voltmeter seem unnecessary as regards the source.

The essential feature of the diaphragm at *d* (Fig. 1) is the restriction of the cone of light to the approximate dimensions of the stellar image under examination. By the use of two such diaphragms used interchangeably, a wide range of magnitudes is covered for any given plate. In practice the diameters of the apertures most generally used were Nos. 60 and 75 on the standard drill gauge, corresponding to about 1.0 mm and 0.5 mm, respectively.

As originally designed, the diaphragms employed were drilled in a sliding strip of brass flush with the stage of the instrument. Since the ordinary commercial plates are slightly concave, it was found that a large and troublesome source of error was being introduced, the effect of the concavity providing a variable distance between the stellar images and the diaphragm. This varied seriously the dimensions of the cone of light transmitted to the thermopile for stars of equal magnitude when located on different parts of the concave surface. After several experiments, a conical design for the diaphragm was adopted, projecting slightly above the level of the stage of the instrument and thus insuring an invariable contact with the film under all cricumstances (Fig. 2). Any error due

to the variation of the slight tilt of the plate thus made necessary was carefully looked for but found negligible.

In the design of the projecting lens two factors have been borne in mind: first, the requisite focal length to produce proper magnifying power within the limits of adjustment; and, secondly, utilization of all the principal rays of the cone emanating from *d*. Change of diaphragm in general will be accompanied by change in the projecting lens in order that the projecting image may cover but not overspread the receiving surface of the thermopile. Two projecting lenses are thus provided corresponding to the two diaphragms most generally employed.

The complete instrument and auxiliary apparatus consisting of galvanometer, resistance box, volt- and amperemeters are shown as in use in Fig. 3.

Adjustments are provided for the light-source in three coordinates. The stage and projecting lens are also adjustable for centering and focusing. A series of diaphragms are provided and may be placed interchangeably upon the stage.

FIG. 2

In the selection of a galvanometer, several factors enter. Theoretically the thermopile is working most efficiently when the resistance of the galvanometer equals its own. For this reason as well as for its high sensitivity, the Thompson type is that usually employed in thermo-electric measurements. On the other hand, convenience of manipulation and freedom from outside disturbances favor the D'Arsonval or moving-coil type whenever such can be adapted to the end sought. Most satisfactory results have been obtained in the present investigation with a Leeds and Northrup galvanometer, Type H, having a resistance of 50 ohms and a sensitivity of 0.75 microvolt. The critical damping resistance of such an instrument is 120 ohms, and is obtained by using external resistance in series with galvanometer and thermopile. With diaphragm No. 60 and a photographic plate of normal density, these conditions afford deflections conveniently large and within the limits of the scale. To obtain corresponding deflections with smaller diaphragms the external resistance is necessarily reduced, increasing the sensitivity of the galvanometer and lengthening its period.

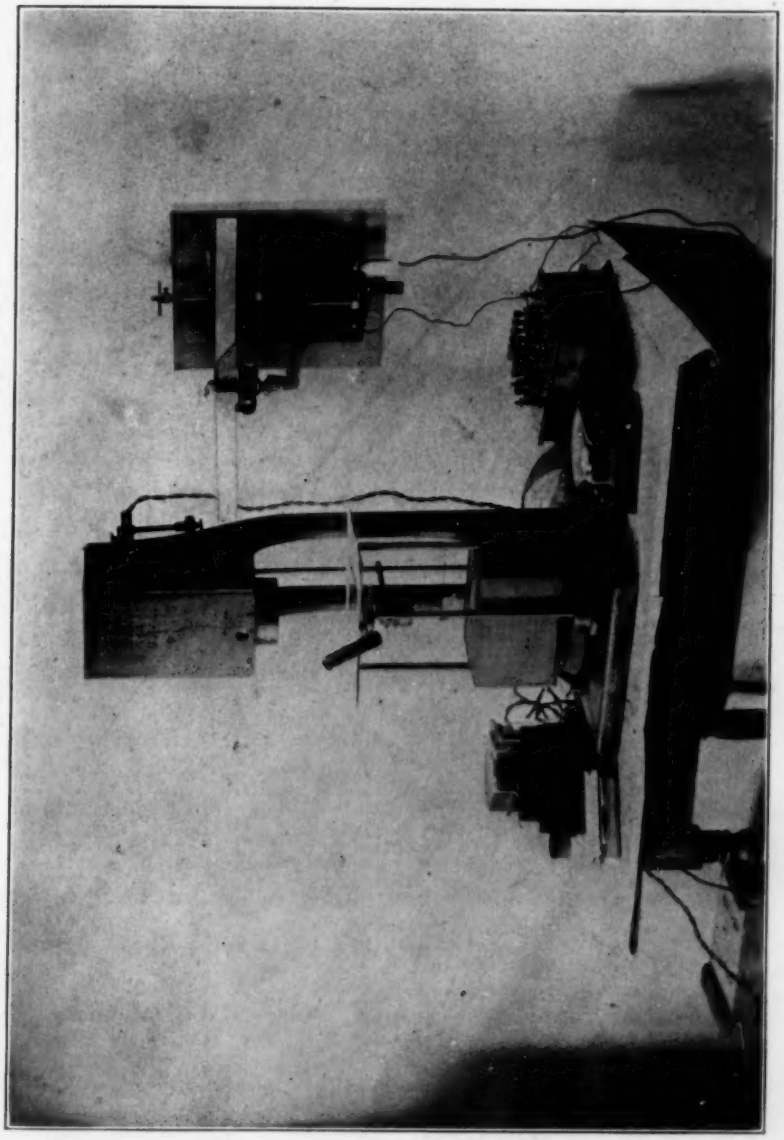


FIG. 3.—The thermo-electric photometer and auxiliary apparatus

As few connections as possible should be made between the galvanometer and thermopile, care being taken to avoid unlike metals, to use ordinary means for maintaining uniform temperature and freedom from draughts, and to protect all binding-posts and connectors with cotton wool.

METHODS OF MEASUREMENT AND REDUCTION

In measuring a plate for reduction, a diaphragm somewhat larger than the stellar images to be measured is selected and put in place upon the stage of the instrument. Since the construction of the diaphragm is such that it projects slightly above the level of the stage-platform to insure contact with the film, the plate is always placed film side down with care. The stellar image in question is placed over the diaphragm, and brought to the center of the latter by viewing its image upon the thermopile by means of the finding eyepiece. As soon as the galvanometer has come to a steady deflection the reading is noted. The stellar image is then moved just off the diaphragm and the reading of the galvanometer again noted as an index of the amount of energy transmitted through the unobstructed portion of the plate and film.

Since the deflections of the galvanometer are taken proportional to the difference of potential impressed, and the difference of potential maintained by the thermopile is directly proportional to the heating effect, then the amount of energy absorbed by the stellar image may be expressed as

$$q=K(D-D'),$$

where D and D' represent the two readings of the galvanometer and K is a constant of proportionality.

Again, since the size and density of the stellar image upon the plate increase with the brightness and hence with the magnitude of the star, which will be denoted by m, for any given exposure we may write:

$$m=f(D-D'),$$

and the problem of reduction is the problem of ascribing the form of the function existing for given conditions. Let A be the area of the free aperture of the diaphragm (Fig. 4). Consider the stellar image which, because of its size and blackness, absorbs energy, as having an effective diameter d equal to that of an opaque disk absorbing the same amount of energy. If ρ = the energy transmitted per unit-area, then we have the relation

FIG.

$$\frac{\rho A}{\rho\left(A-\frac{\pi}{4}d^2\right)}=\frac{D}{D'} \tag{1}$$

where D and D' represent the galvanometer-deflections corresponding to the plate-readings taken with free aperture, and with a stellar image interposed, respectively. The above may be simplified and written

$$\frac{A}{\frac{\pi}{4}d^2}=\frac{D}{D-D'} \tag{2}$$

whence

$$d^2=\frac{4A}{\pi}\left(\frac{D-D'}{D}\right)=c\delta \tag{3}$$

where δ represents the difference in deflection between the free and image-readings for a given star per centimeter of deflection for free aperture, and c is the constant of proportionality.

If we assume the well-known relation between magnitude and measured diameters to hold in the present instance, we have

$$m=a-b\sqrt{d},$$

or

$$d=\left(\frac{m-a}{b}\right)^2.$$

Hence, substituting in (3), we obtain

$$\left(\frac{m-a}{b}\right)^4=c\delta$$

$$m-a=bc^{\frac{1}{4}}\delta^{\frac{1}{4}};$$

or, writing new constants,

$$m=\alpha-\beta\delta^{\frac{1}{4}}. \qquad (4)$$

This formula, though indirectly derived from the square-root formula, stands merely as an empirical expression in an attempt at the solution of $m=f(\delta)$. It will be observed that in defining δ as the ratio $\frac{D-D'}{D}$ no assumption as to the constancy of D is involved. The value of D will depend upon the transparency of the plate-film, thickness of the glass, voltage of the source, resistance in the galvanometer-circuit, etc.

Since the reduction involves only the ratio $\frac{D-D'}{D}$, it will be noted that a change in D due to a change in intensity will not affect the results, provided conditions remain constant throughout the measurement of D and D' for a given star. In like manner, variation in the transparency of glass and film over different parts of the plate will be largely if not wholly eliminated, since an increase in absorption though reducing the deflections does not alter this ratio. This was verified experimentally by taking readings on stellar images on a given plate in the usual manner, then remeasuring the same stars with absorbing media interposed in the path of light to the thermopile. Although D was reduced about 50 per cent, the ratio $\frac{D-D'}{D}$ remained the same for each star within the limits of accidental errors. Similar results were obtained from measurements of the same stellar images for different values of voltage.

The first plates reduced were those taken at the Dearborn Observatory, with the 18-inch refractor, a color-filter, and Cramer Isochromatic plates. It was known from careful measurements that the square-root formula gave good results for measured diameters on these plates.

The close approach to a straight-line relation between magnitudes of the Pleiades and the fourth root of the galvanometer-deflections obtained by measuring images on the thermopile is shown graphically in Fig. 5, and justifies the reduction-formula experimentally. Müller and Kempf's[1] values for the magnitudes

[1] *Astronomische Nachrichten*, **150**, 203, 1899.

are laid off as abscissae, and the fourth root of the corresponding galvanometer-deflections as ordinates. The constants α and β of (4) may be taken at once from the graph, and used for the determination of any unknown magnitude on the plate.

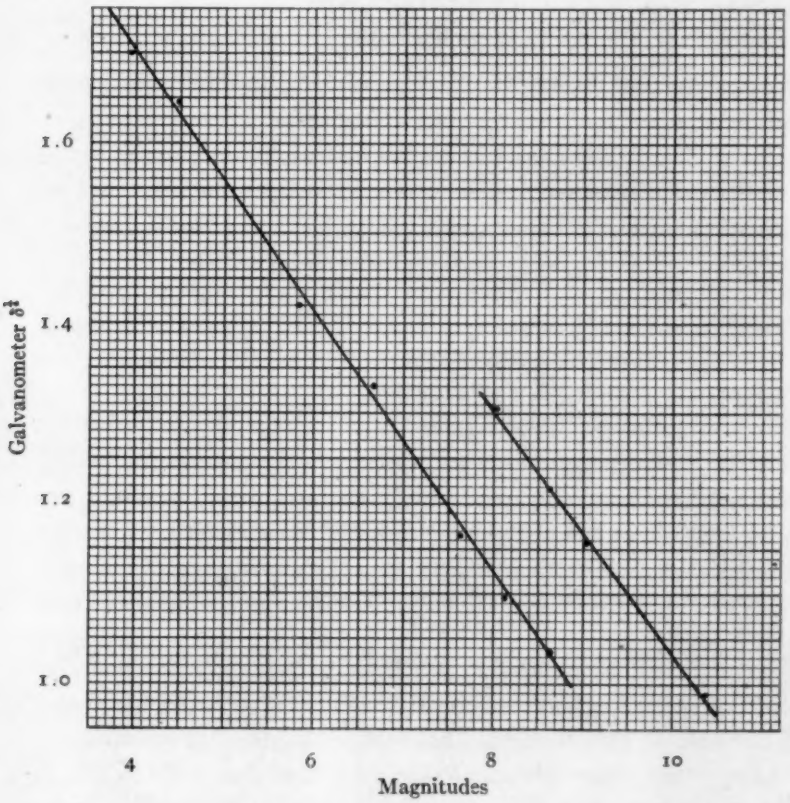


FIG. 5.—From Plate A 140 taken with 18-inch refractor and color-filter at Dearborn Observatory.

As the stellar image becomes smaller relative to any particular diaphragm, the proportional amount of energy absorbed becomes less and the consequent change in the galvanometer-deflection decreases, as is evident from the algebraic expression $m=\alpha-\beta\ \delta^{\frac{1}{4}}$. For small values of δ the change in $\delta^{\frac{1}{4}}$ becomes less appreciable and therefore this increase of error is compensated for by substituting a smaller diaphragm and proceeding as before.

If Δm represents a known difference in magnitude between two stars, and $\Delta\delta^{\frac{1}{4}}$ the corresponding difference in $\delta^{\frac{1}{4}}$ for the two readings, then, for any given diaphragm, the slope of the straight-line relation becomes

$$\frac{\Delta m}{\Delta\delta^{\frac{1}{4}}}=\beta.$$

For the same Δm measured with a second diaphragm we have similarly

$$\frac{\Delta m}{\Delta\delta'^{\frac{1}{4}}}=\beta',$$

whence the relation between the slopes of the two plots becomes

$$\frac{\beta}{\beta'}=\frac{\Delta\delta^{\frac{1}{4}}}{\Delta\delta'^{\frac{1}{4}}}, \tag{5}$$

and readings taken with any diaphragm may be reduced to the corresponding scale of any other when two stars are measured in common with the two diaphragms.

In Fig. 5 the graphs are drawn separately for each diaphragm on the same scale and indicate clearly the use of the smaller aperture for the fainter magnitudes. It should be added that with the substitution of the smaller diaphragm, the sensitivity of the galvanometer was increased by lowering the external resistance from 120 to 40 ohms.

In practice it is found that a single diaphragm may be relied upon for a range of three or four magnitudes in measurement, and that the use of two different diaphragms is usually sufficient to cover the entire range of measurable magnitudes on a single plate.

To understand more fully the rôle of the diaphragm and conditions of sensitivity in measurement, let R be the radius of any given aperture used for the diaphragm, and r the effective radius of the stellar image. From Fig. 4 and equations (1) and (2) we may write

$$\frac{\pi R^2}{\pi r^2}=\frac{D}{D-D'},$$

or, writing δ for $\frac{D-D'}{D}$ as before, we have

$$\delta=\frac{r^2}{R^2}; \tag{6}$$

whence by differentiation

$$\frac{d\delta}{dr}=\frac{2r}{R^2}. \tag{7}$$

The sensitivity of the arrangement represented by the derivative $\frac{d\delta}{dr}$ is therefore directly proportional to r, since R is constant for any given diaphragm.

A varying degree of sensitivity dependent upon the size of the image might seem at first a serious and troublesome disadvantage. As will be shown, however, the difficulty is more apparent than real, and may be controlled within limits usually exceeded by accidental errors.

As the sensitiveness of the instrument falls as a result of small values of r relative to R, a new diaphragm of smaller dimensions is substituted. Reference to equation (7) will show that such a change causes an increase in the sensitivity, inasmuch as $\frac{d\delta}{dr}$ varies inversely with R^2, although the free deflection D will be less than before in consequence of the corresponding change in the amount of energy admitted to the thermopile. This change in D, however, may be conveniently compensated for by decreasing the resistance in the galvanometer-circuit until D regains its former value. Although theoretically $\frac{d\delta}{dr}$ depends only on r and R and is quite independent of the magnitude of D, nevertheless in practice it will be seen that the accuracy with which δ may be observed is greater for large values of D. For this reason the resistance in the galvanometer-circuit is always adjusted to maintain D conveniently large for a given set of readings.

Let us suppose for the sake of comparison that we were to measure transparent stellar images upon an opaque ground, such as would be the case in measuring a glass positive instead of the original negative. In this case the galvanometer-deflection D, due to the energy transmitted by the stellar image, would be directly proportional to the area of the image or to r^2, in the form

$$D=Kr^2,$$

the derivative of which would indicate the sensitiveness of the arrangement

$$\frac{dD}{dr}=2Kr.$$

Comparing this expression with equation (7) above, we see that as before the sensitiveness varies directly with the radius of the image without the advantage of the controlling factor R^2. Experimental tests with both positives and negatives revealed at once the increased sensitiveness to be obtained with the use of the interchangeable diaphragms on negative plates.

Aside from the advantage of using the original plate, moreover, the ease of manipulation in locating and setting upon the several stellar images is much facilitated by using the plates with dark images against a clear background, as is the case in the negative form. Again, since readings taken from positive plates would be subject to uncertain corrections for opacity of the background and for varying degrees of transparency of glass and film, the advantage lies in the clear glass background, where the condition of the film is constantly under observation and the galvanometer-deflections are large.

Since the sensitiveness of the thermo-electric photometer, if we may so term the apparatus, has been shown to be a function of r for any particular diaphragm, it becomes important to investigate the changing effect of error in δ upon the resulting magnitude.

Making use of the equation of reduction (4),

$$m=a-\beta\delta^{\frac{1}{4}},$$

and differentiating, we obtain

$$\frac{dm}{d\delta}=-\frac{\beta}{4\delta^{\frac{3}{4}}}.$$

If the galvanometer-deflections are recorded in centimeters, the value of β for the Dearborn plates is very nearly 6.5. The same is true for Cramer plates and color-filter used with the 6-inch Zeiss doublet of the Yerkes Observatory. If we assume for the error of the mean of several settings on a star an accidental error not

greater than $\Delta\delta = 0.01$ cm, a very plausible assumption, we may express the resulting error in stellar magnitudes as

$$\Delta m = \frac{6.5 \times 0.01}{4\delta^{\frac{3}{4}}} = 0.0162\ \delta^{-\frac{3}{4}}.$$

A curve was therefore to be platted for $\beta = 6.5$ using values of δ as abscissae and the corresponding computed value of Δm as ordinates (Fig. 6). Similarly, a family of such curves could be constructed

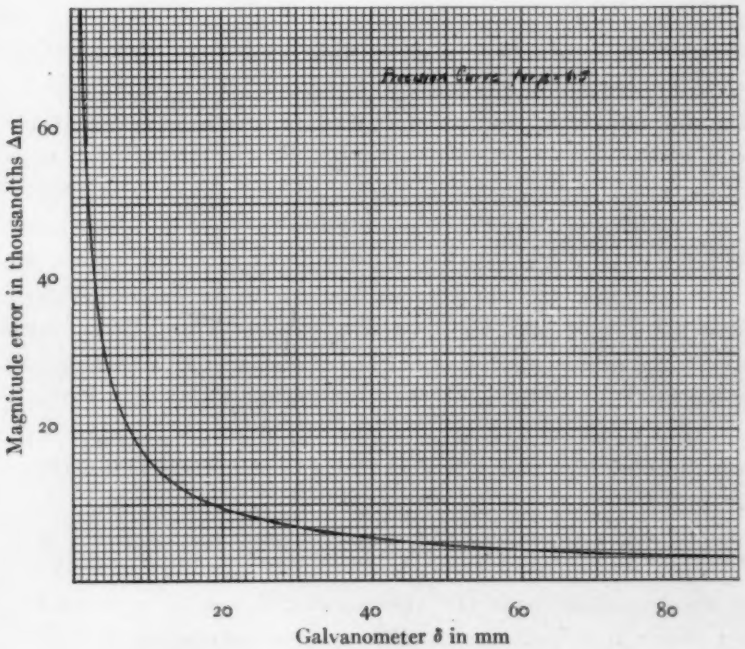


FIG. 6.—Precision curve for $\beta = 6.5$

with the parameter β. Such a curve serves at once to indicate the degree of precision attainable with given conditions of plate, galvanometer, and diaphragm. As will be seen, for example, when δ, the fall in galvanometer-deflection due to the interposition of the stellar image, is equal to 1 cm, the error in magnitude resulting from a reading-error of 0.01 cm, will be $0^{M}016$ and will remain less than $0^{M}01$ if δ does not fall under 1.6 cm. Unfortunately, systematic

errors due to conditions of atmospheric transparency or irregularities in "seeing" and plate-sensitiveness seldom warrant higher precision in measurement, though such should be easily obtainable if the stellar images are not too minute to afford larger values of δ. Except in the case of the faintest images, readings are seldom made when $\delta < 1$ cm. When, therefore, in a series of measurements δ reaches the limiting value, a smaller diaphragm is substituted, and the sensitiveness of the galvanometer increased by lowering the variable resistance, with the resulting increase in δ for the same star.

In the actual reduction of plate-magnitudes and the platting against a known or assumed scale of magnitudes as is shown for the Dearborn plate in Fig. 5, care must be taken to differentiate between errors of measurement and systematic errors often neglected, such as correction to the center of the plate, color-curve of the lens, and the relation of color-equation to plate and filter used with a given outfit. As many of these quantities are at best uncertain, it is seen that we could hardly expect a given plate to show residuals not larger than those attributable to errors of measurement. In the interpretation of the curve in Fig. 5 it is to be noted that no correction to the center was attempted, and the approximation to the straight line indicates the degree of agreement between the color-filter and the Cramer plate combination, and the Müller and Kempf "eye."

MEASURES OF THE MAGNITUDES OF STARS OF THE PLEIADES

In investigations with the thermo-electric photometer at the Yerkes Observatory, plates have been measured, taken with each of the several instruments regularly in use, the 6-inch Zeiss camera, the 2-foot reflector, and the 40-inch refractor. Fig. 7 shows the results of platting measurements of the Pleiades in precisely the same manner as was done in Fig. 5. Inasmuch as the field of the Zeiss doublet shows no appreciable correction for distances less than 0.05 from the center,[1] this somewhat uncertain quantity may be considered as inappreciable in the case of the Pleiades plates where the group is centrally located.

[1] *Astrophysical Journal*, **36**, 179, 1912.

In this comparison the agreement with Müller and Kempf's values is at once apparent. The value of the constant β of the reduction-formula as determined by the graph is 6.49 against 6.50 for the Dearborn plates.

To indicate clearly the method of measurement and reduction as well as the amount of agreement among individual settings, a specimen reduction-sheet is shown for Plate UV 1261 (Table I).

The first column gives the Bessel designation of the star. The second column gives the galvanometer-readings in centimeters, when the stellar image is located centrally on the diaphragm, and

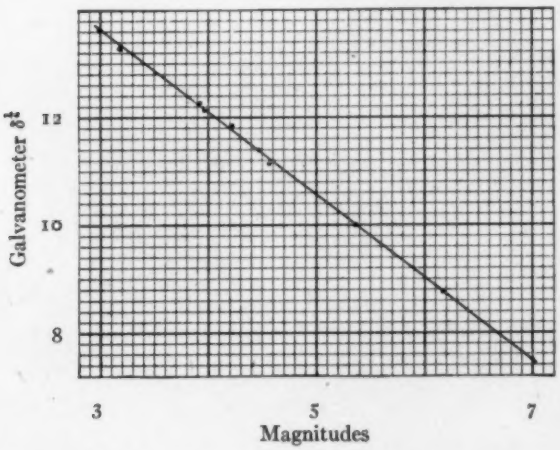


FIG. 7.—From Plate UV 1261, taken with Zeiss doublet and color-filter at Yerkes Observatory.

the third column shows the readings corresponding to the free film of the plate in the immediate neighborhood of the image. Three settings are usually taken as sufficient for each star, and the mean is used. The plate-readings serve in each instance to check any progressive errors in the transparency of the film or glass as well as the effect of possible change in the voltage of the light-source. Although a slight fogging of the background seems to have little effect upon the size or blackness of the stellar image, it enters of course with full value upon the transparency of the background with which each image is compared, and so needs to be eliminated. To

TABLE I

PLATE* UV 1261, PLEIADES. MEASURED DECEMBER 17, 1914

Volts, 51.5. Amperes, 3.8. Galvanometer Resistance, 40. Diaphragm No. 75

STAR	GALVANOMETER DEFLECTION			δ	$\delta^{\frac{1}{4}}$	MAGNITUDE		
	D'	D	$D-D'$			Müller and Kempf	Computed	Residual
η	9.83 cm	14.20 cm						
	9.80	14.21						
	9.75	14.16						
	9.79	14.19	4.40 cm	3.10	1.326	3.19	3.23	+.04
f	10.88	14.10						
	10.94	14.08						
	10.90	14.08						
	10.91	14.09	3.18	2.26	1.225	3.92	3.90	−.02
b	10.83	13.80						
	10.83	13.88						
	10.88	13.81						
	10.85	13.83	2.98	2.16	1.212	3.96	3.98	+.02
c	11.40	14.18						
	11.38	14.19						
	11.39	14.18	2.79	1.97	1.185	4.21	4.16	−.05
d	11.56	13.89						
	11.58	13.99						
	11.69	14.04						
	11.61	13.97	2.36	1.69	1.139	4.48	4.46	−.02
e	11.02	13.10						
	11.00	12.95						
	11.00	12.91						
	11.00	12.98	1.98	1.53	1.114	4.57	4.62	+.05
h	12.69	14.12						
	12.70	14.10						
	12.70	14.11	1.41	1.00	1.000	5.38	5.36	−.02
k	12.78	13.53						
	12.79	13.59						
	12.78	13.56	0.78	0.58	0.874	6.17	6.18	+.01

$\beta = -6.50$ $\alpha = 11.86$ p.e. $= \pm 0.022$

* Cramer Instantaneous Isochromatic with visual luminosity filter β 10.

avoid decimals it is found convenient to take for δ ten times the ratio $\frac{D-D'}{D}$. The quantity δ may now be defined as the difference in galvanometer-deflections between the stellar image and its background for a standard plate-reading of 10 cm. Column five therefore gives

$$\delta = \frac{D-D'}{D} \times 10.$$

The fourth root of δ is entered for each star in column six and is easily obtained by use of a table of squares and square roots of numbers. If the number whose fourth root is desired is located in the column marked "Square," its fourth root may at once be taken from the column marked "Square Root"—such tables for numbers from 1 to 1600 are usually found in the numerous engineers' handbooks and require little interpolation.

The last three columns give the magnitudes found by Müller and Kempf, the magnitudes computed from the constants $\alpha = 11.86$, $\beta = -6.50$, obtained from the least-square solution, and the resulting residuals, indicating close agreement between the magnitudes photo-visually determined with the Zeiss camera and Müller and Kempf's scale, the probable error for a single star being $0^{M}.022$.

Ordinary plates taken without light-filter with the 2-foot reflector of the Yerkes Observatory were reduced in a similar manner and justified again the use of the fourth-root law. The results of reducing Pleiades-magnitudes from Plate R 531 is shown graphically in Fig. 8. The photographic magnitudes platted as abscissae are those of Münch determined at Potsdam. The solid dots indicate the reduced magnitudes after correction to the center of the plate. Although the aperture of the mirror was reduced to 12 inches for the particular exposure in question, the field is still subject to correction for distances greater than $\rho = 4'$ and amounts to $0^{M}.2$ or more at a distance of $\rho = 20'$. The method of calibration of this field correction was similar to that previously described in this *Journal*[1] and needs no detailed description here. In the present instance Plate R 2594 was selected, which contained four sets of images taken across the optical axis. The absorption of these images was

[1] *Astrophysical Journal*, **38**, 224, 1913.

measured with the thermopile in the usual manner and the $\delta^{\frac{1}{4}}$ corresponding to each image was platted against the distance of the images from the center of the plate, expressed in minutes of arc. Fig. 9 shows the resulting graph. The departures from the mean

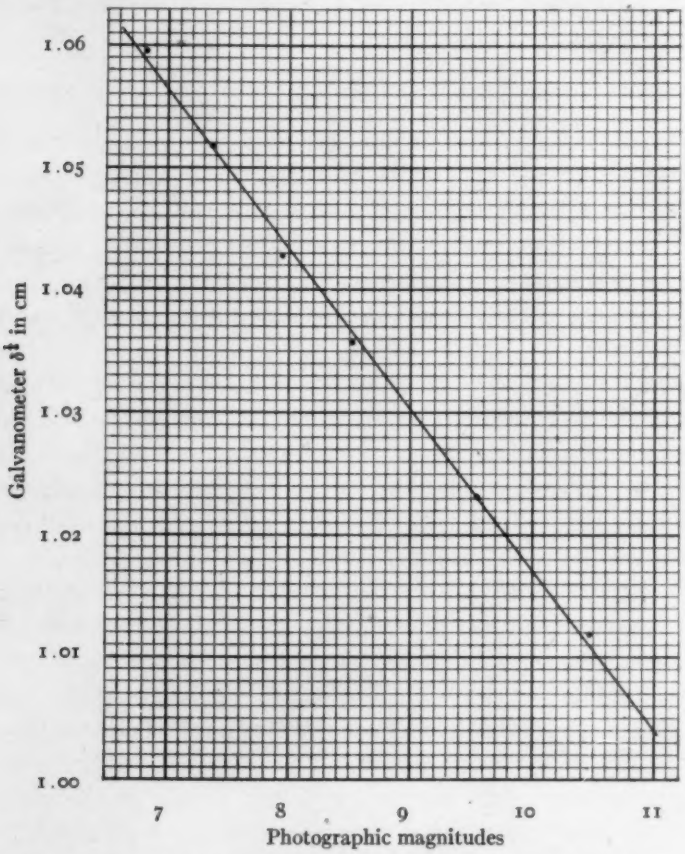


FIG. 8.—From Plate R 531, taken with 2-foot reflector at Yerkes Observatory

curve may be attributed to changes in "seeing," atmospheric transparency, and small errors in the equality of the times of exposure. These curves are all similar and express the variation of $\delta^{\frac{1}{4}}$ with distance from the optical axis for four different-sized images corresponding to four different values of $\delta^{\frac{1}{4}}$ for any given distance from the axis. Within the range of images measured, no particular

dependence of the form or size of correction upon the size of the image seems apparent. It must be admitted that corrections made in this manner seem at best artificial, and at present it seems exceedingly doubtful if corrections taken from certain plates used experimentally can be applied generally to any other plate without large assumptions as to the order of accuracy maintained. In applying corrections from the curves of Fig. 9 to the magnitudes of R 531, Fig. 8, the constant β of equation (4) was so chosen as to reduce to a minimum the residuals of the corrected magnitudes of R 531.

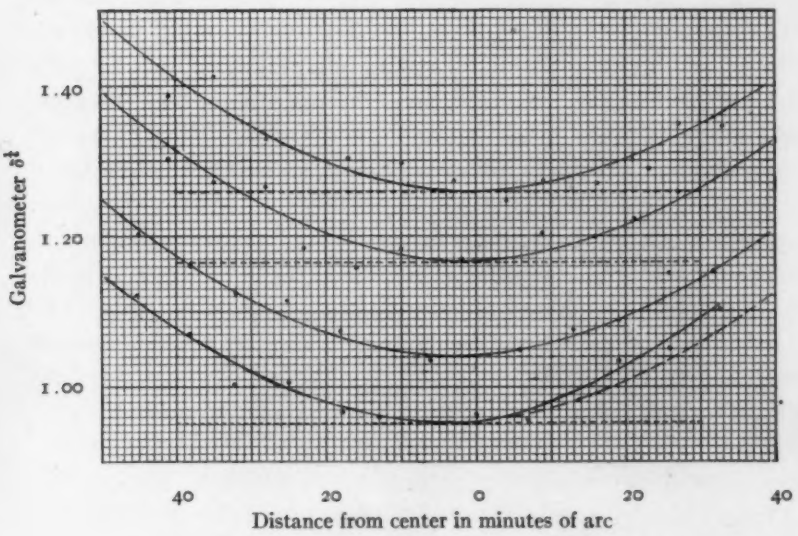


FIG. 9.—Calibration curves for 2-foot reflector from Plate R 2594

This gives a value of $0^{M}.026$ for a change in $\delta^{\frac{1}{4}}$ of 0.01 unit. Stars 24 and 27 (Bessel) receive the largest corrections, being 17′ and 26′ from the axis, respectively.

The whole matter of finding suitable methods of correction to the center of the plate is one of the most obstinate difficulties in the way of exact photometry on an absolute scale. A high degree of precision in the measurement of photographic images will therefore best be rewarded in the investigation of relative magnitude-changes or in the photometry of small areas where the field under investigation is restricted to the vicinity of the optical axis.

Other difficulties in the way of more accordant results from various observers is the lack of uniformity of equipment and spectral sensitiveness of the plates used. "Photographic magnitudes," as at present published are understood to be the results of measurement of the stellar images on "ordinary" (blue-sensitive) dry plates exposed at the focus of a reflector or a photographic refractor (corrected to blue rays). Unfortunately the difficulties of non-uniformity in color-sensitiveness of the various dry plates on the market is greatly accentuated by the wide divergence in the optical equipment employed. Unless a color-filter is used, the uncorrected, and out-of-focus rays will materially affect the size and blackness of the stellar image on the plate. Moreover, the effect will be quite different for differing spectral types, the difficulty being exaggerated for focal settings on either side of the normal position. Again, a photographic objective of special Jena glass will undoubtedly behave somewhat differently from a photographic lens of the usual flint and crown combination. In the case of the reflector, where radiations of all wave-lengths are combined, the effect must invariably differ from the case of any refracting telescope. These divergences, small or large, will become more apparent with greater refinement in the measurement of the photographic images and indicate further fields of investigation in stellar photometry.

It would seem that, for agreement on a precise scale for magnitudes, the reflector would be the ideal instrument for photography, as the question of a color-curve would thus be entirely eliminated so far as the instrument is concerned. The chief difficulty remaining would then be the adoption of a standard plate of given spectral sensitiveness, or, in want of such plates, employing a prescribed filter of known absorption which would produce as far as possible a certain effect for a given brand of plates.

For purposes of completeness in the application of the fourth-root law for the reduction of thermopile-measures, tests were made upon plates taken with the 40-inch refractor through color-filter and with isochromatic plates. Fig. 10 shows the magnitudes of the Pleiades on Müller and Kempf's scale platted against the corresponding $\delta^{\frac{1}{4}}$ of the galvanometer-readings. From other series of measurements covering a variety of times of exposure, the slope of

the line appears quite independent of the time of exposure, at least within the range of exposures used.

Although considerable unpublished material exists relative to the magnitude-corrections to the center of the plate for the 40-inch refractor, and further investigations were made using the thermopile method for plate measurement, consistent results have not been obtained which may be trusted within the degree of precision

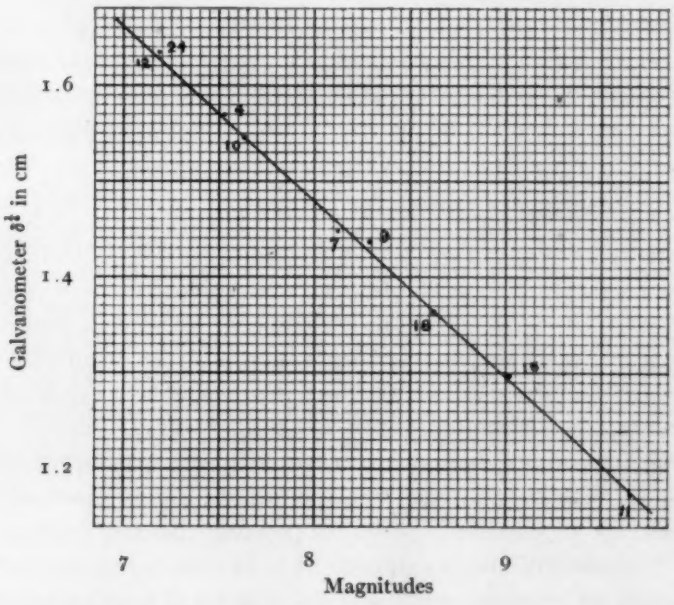


FIG. 10.—From Plate 015, taken with 40-inch refractor and color-filter at Yerkes Observatory.

desired. Owing to rapid and uncertain changes in seeing and the great focal length, it has been found practically impossible to use the simple method employed for the calibration of the reflector plates. Inasmuch as the stars of the Pleiades platted above cover a considerable area on the plate, the residuals indicated are doubtless attributable to this troublesome and uncertain correction to center which has yet to be applied. A still further difficulty in the way of applying this correction is the uncertainty of locating with exactness the position of this so-called "center." Although a rough

approximation for the position of the optical axis with a given plate may be obtained from the intersection of the prolonged major axis of the elongated stellar images near the edge of the plate, the errors introduced by the uncertainty in the location will easily amount to several hundredths of a magnitude at large distances from the axis. Further, the stability of the position of this "center" from plate to plate should not be assumed without careful investigation.

The investigations of Schlesinger[1] in locating the optical axis in the case of the 40-inch refractor, and the subsequent precautions taken in obtaining plates for stellar parallax, may not be without application to work of precision in the line of photographic photometry.

SOURCES OF ERROR IN THE PLATE

Enough has already been said in regard to sources of error for which the telescope is responsible. To a large degree these errors are susceptible of evaluation and correction, if not of elimination. There remains, however, one serious source of error in all photographic photometry which has its effect in purely relative measurement as well as in determinations on a standard scale. Such is the plate itself. Residual errors due to a lack of uniformity in the sensitiveness and thickness of the emulsion on the plates must exist, and it remains to eliminate as far as possible dangers from this source. Fortunately the magnitude of such errors has not been large enough to interfere seriously with the tenth's place on the magnitude-scale, and for this reason such errors have in general aroused little attention. If, however, we hope to push forward another decimal place in astronomical photometry, here lies one of the fields for further investigation and research.

Neglecting the region of the plate within 1 cm from the edge, Parkhurst[2] found from several hundred measures on Seed and Cramer plates at the Yerkes Observatory that the effect of local variations in sensitiveness of film would in general be less than $0^{M}.04$, and under most unfavorable circumstances might rise to $0^{M}.1$.

[1] *Astrophysical Journal*, 32, 376–378, 1910.

[2] *Astrophysical Journal*, 31, 20, 1910.

This error could doubtless be much reduced by employing plates especially coated on plate glass. The concavity of ordinary commercial plates introduces another source of error, and has been investigated by Parkhurst, who estimates the probable effect for focal measures to be of the order of $0^{M}.01$ or $0^{M}.02$.

Investigations by Hartmann[1] reveal somewhat larger sources of error than those found by Parkhurst.

In a further study it is purposed by means of the thermopile to investigate extensively a large number of plates of both domestic and foreign manufacture on both commercial and plate glass to determine, if possible, the maximum degree of precision which may be relied upon in the use of the plates now upon the market. When one considers the extreme sensitiveness of the photographic plate as a detector of radiation, and the possibility of obtaining measurable images of faint stars hopelessly beyond the range of any other known "radiometer," the importance of perfecting such a medium for work of precision cannot be overemphasized. If the cooperation of observatories and other scientific institutions using large numbers of plates could be secured, it would doubtless be possible through special manufacture to secure plates much superior to any of the commercial brands now upon the market.

METHOD OF EXTRA-FOCAL MEASUREMENT

The adaptation of the thermo-electric photometer to the direct measurement of plate-opacities is at once apparent. For such work, since the area of the region to be measured is presumably more than sufficient to cover the diaphragm, a large aperture may be used and the galvanometer-resistance reduced for greater speed of manipulation. For such work galvanometer δ's are taken as directly proportional to plate-opacities. If extra-focal stellar images are being measured, correction for a lack of uniformity in the absorption of the glass can be made by taking readings on the free glass in close proximity to the image, as is done in the measurement of focal plates, already described. Here, as in the other case,

[1] "Ueber die Konstanz der Empfindlichkeit innerhalb einer photographischen Platte," *Eders Jahrbuch für Photographie*, 1906.

precision of results rests, first, on the assumption of uniform sensitiveness of the photographic film, and secondly, on the assumption that the absorption of the glass for the region immediately surrounding the stellar image is the same for the region beneath the image under measurement.

The advantage which the thermopile has over the "microphotometer" for extra-focal work is the elimination of the personal equation by the shifting of responsibility for correct judgment of opacities from the eye to a purely mechanical device. The order of accuracy of setting is about the same for the two methods, though, of course, the results of the Hartmann method are influenced by the skill of the observer.

To illustrate the degree of blackening due to various exposures, a plate containing a series of "standard squares," produced by simultaneous exposures to light-sources of known ratio of intensity, was measured with the thermo-electric photometer. The graph in Fig. 11 shows the relative blackening as measured by the galvanometer-deflections platted against the relative intensities expressed in magnitude-differences. Since the exposures were all made simultaneously, the validity of the reciprocity law $E=It^p$ does not enter, the values of E being to each other as the intensities of the source. It should be stated that the plates used for this test were selected from among those used by Parkhurst for another purpose and described by him in an earlier publication.[1]

Readings were taken upon the clear background of the plate adjacent to each image as usual, and subtracted from the galvanometer-readings for the image alone. Assuming that the galvanometer-deflections so corrected will be directly proportional to the transparency of the image, we should expect to find a nearly linear relation between the deflections so recorded and the logarithm of the "exposure" E, or a direct proportion between the quantity E as expressed in magnitudes. Data concerning the several apertures admitting the light to the several images have already been published (*loc. cit.*), together with the relative magnitudes. It will be observed that an approximate relation holds over a range of about one magnitude only.

[1] *Astrophysical Journal*, 26, 244, 1907.

An extra-focal plate (blue-sensitive) of the Pleiades taken with the Zeiss camera was then measured with the thermo-electric photometer. The graph obtained from platting magnitudes against the logarithm of the galvanometer-deflections is shown in Fig. 12. The bending of the curve for the brighter stars due to overexposure

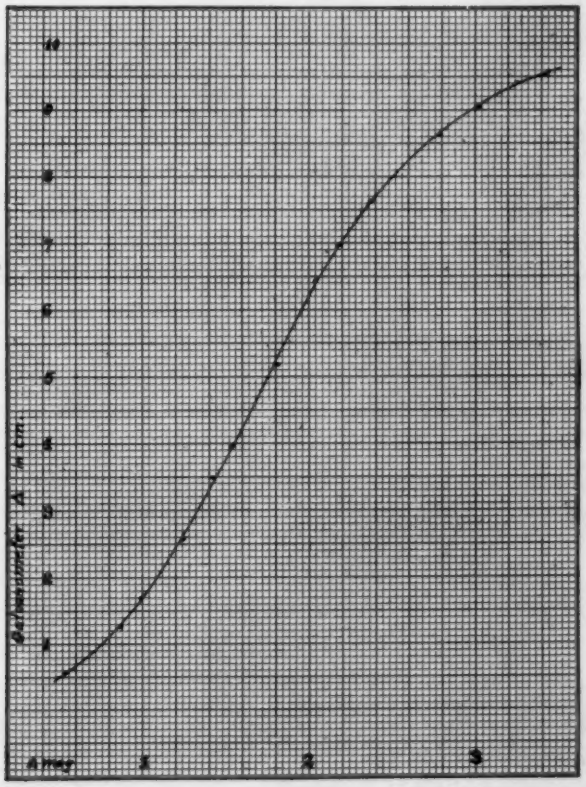


FIG. 11.—Curve showing plate opacities in terms of galvanometer deflections

indicates the comparatively narrow limits within which the law of blackening may be applied. In extra-focal work several exposures properly timed will ordinarily be needed to cover the desired range for a given field. The same limitation applies of course equally well to measurements of extra-focal images by any other method.

SPECTRO-PHOTOMETRY

The idea of measuring relative intensities in spectral plates by means similar to the foregoing is not new. The extended investigations of Koch[1] resulting in the so-called "self-registering" microphotometer have proved not only the feasibility of the method but

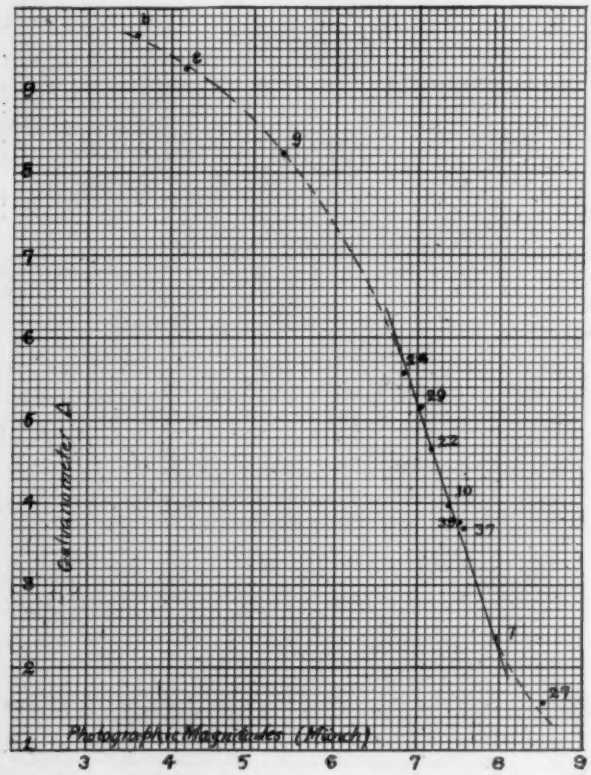


FIG. 12.—Extra-focal measures of magnitudes of stars of the Pleiades (the designation is that of Bessel).

the valuable results which may be obtained by such methods of measurement, especially in the study of laboratory spectra.[2] The apparatus under present consideration, when provided with a

[1] *Annalen der Physik*, **39**, 705, 1912.

[2] *Astrophysical Journal*, **39**, 213, 1914.

micrometer-screw stage, becomes at once adapted to such measurements. When so used, the small circular diaphragm is replaced by a small adjustable slit, which all but touches the under and film-side of the spectral plate to be measured. Arrangement is then made whereby a sliding platform moved by an accurately cut micrometer-screw may be attached to the main stage of the instrument. The spectral plate is held to the latter by clips in the usual manner, with the film down and in close contact with the slit-diaphragm. The spectrum is then exposed by setting the micrometer-screw for a given reading and noting the galvanometer-deflection corresponding to the opacity of the negative, then advancing the plate a known fraction of a millimeter, and recording the corresponding galvanometer-reading against the screw-setting, continuing the process till the region in question has been covered. This arrangement, though lacking the elegance of the "registering micro-photometer," has the advantage of simplicity, the thermopile replacing as detector the somewhat more troublesome photo-electric cell as used in the more elaborate apparatus of Koch.

In the case of objective-prism photographs, when the linear dimensions of the stellar spectra are not great, the task of mapping the intensity-curve is not difficult and may be accomplished in a comparatively short time. After the spectrogram has been properly lined up on the stage no further attention is necessary other than noting the reading of the galvanometer after each turn of the screw. Results are shown for measurements of an objective-prism plate of the Pleiades taken with the Zeiss ultra-violet doublet previously mentioned. The graphs in Fig. 13 show the intensity-curves for the photographic spectra of the three stars Alcyone, Taygeta, and Merope. Screw-readings are platted as abscissae against the direct readings of the galvanometer as ordinates, the plate being advanced 0.01 mm for each setting. The plate for which the curves are drawn is an enlarged positive, the hydrogen lines appearing dark and accordingly represented in the graph by depressions corresponding to the lower values of the galvanometer-reading. The slit is purposely adjusted somewhat wider than the absorption lines in order to minimize their effect and give a somewhat smoother form for the

entire curve of distribution of energy. No shift in the point of maximum radiation is apparent, but the center of gravity of the area formed by the curve and its horizontal base-line is appreciably nearer the violet end for Alcyone than is the case for the other two stars. This would seem to indicate that the bright star is relatively somewhat "bluer" than the other two in question. Careful investigations of spectral types in this manner may afford a more refined

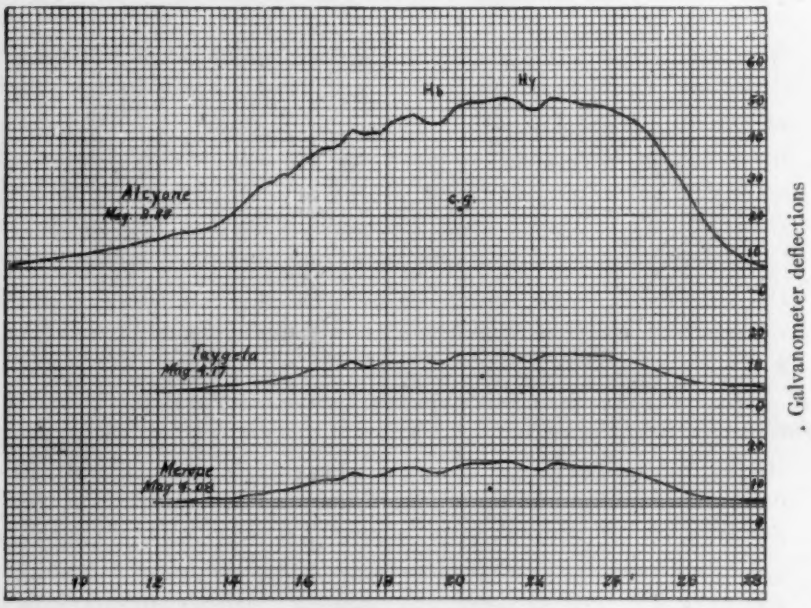


FIG. 13.—Spectral intensities of three stars of the Pleiades

method for establishing color-indices and make possible important quantitative measurements in spectro-photometry.

By narrowing the slit in the case of the arrangement described above, it becomes possible to determine with considerable degree of accuracy the mean position of broad and diffuse lines, which are so difficult of measurement by the usual methods. Further, by a special modification of the design of the thermopile it should become possible to measure with considerable rapidity the location of a

given line, if its spectral intensity is not a question of importance. Such a modification would form an important accessory to the instrument and would prove of special value in the determination of the radial velocities of stars of early type.

A continuation of this subject of thermo-electric methods of measurements, with an application to variable stars, will appear in a subsequent number of this *Journal*.

YERKES OBSERVATORY
March 1916

THE NATURE OF THE CONSTANT-ERROR TERM FOUND IN THE DETERMINATION OF THE SOLAR MOTION FROM RADIAL VELOCITIES

By C. D. PERRINE

INTRODUCTION

I have stated the suspicion that the term K found in the determination of the solar motion from radial velocities was in reality not a constant term but that it was due to unrepresented motion. That suspicion resulted from the finding of peculiarities in the residual velocities in the various spectral classes in different regions of the sky. It is highly important to know whether this term is in reality some form of constant error (using this expression in a broad sense) or whether it is due to motion.

The term K is in effect the *average algebraic residual after taking out the solar motion*. If there is an excess of positive or negative velocities after eliminating the solar motion—whether this excess is due to a few large or many small velocities or whether to a few or many isolated regions of the sky—this excess divided among the stars used will give a constant term. The consistency with which such a constant residual is shown in different parts of the sky should be a good test of the reality of the phenomena as a constant error or physical effect of some kind.

ELIMINATION OF THE SOLAR MOTION

It seems desirable to consider for a moment the reality of the so-called solar motion, elementary though it be. If we consider the evidence afforded by the proper motions, as well as the generally good agreement of the values of the solar motion derived from such heterogeneous data as the widely different average velocities of the different spectral classes and in turn the consistency of the velocities within these spectral classes after the elimination of the solar motion, there seems no reason to doubt that the so-called solar motion is in reality an actual relative translation of the sun among the naked-eye stars. As to what the real motion may be of this mass of stars with respect to others or to the spiral nebulae, plane-

tary nebulae, globular clusters, or other super-systems, we are not concerned for the present.

The question of what value of the solar motion is to be used for any special investigation must depend upon the nature of that particular investigation, whether the point of reference is the center of gravity of a certain class or group of stars, or of all the stars. In the present and similar general investigations it appears preferable to use a common motion rather than the values found for each class separately.

There is some uncertainty in regard to the position of the apex of the solar motion. It is, however, very unlikely that the uncertainty is great enough to affect the general conclusions contained in this paper. The velocity seems even less doubtful. My own investigations convince me of the general correctness of Campbell's conclusion that the velocity of the sun is close to 19.5 km per second in a direction not very far from $18^h, +30°$, referred to the stars which we are discussing.

The observed velocities of the stars have been freed from the solar motion on this hypothesis. Owing to the undoubted connection of some of the principal systematic motions of the stars with the Milky Way, the first investigation related to it. The comparison of galactic and non-galactic regions also seems an efficient test, as these regions appear to be but little affected by uncertainties in the amount and direction of the solar motion.

The residuals were first grouped for the galactic and non-galactic regions separately and the results are given in Table I.

TABLE I

Class	Galactic			Non-Galactic			R_1-R
	R	Mean V	No. of Stars	R_1	Mean V	No. of Stars	
	km	km		km	km		km
B–B5.....	+5.2	8.3	163	+4.3	5.4	17	−0.9
B8–B9....	+1.1	6.0	31	+1.2	6.5	14	+0.1
B all.....	+4.5	8.0	194	+2.9	5.9	31	−1.6
A.....	−1.4	12.2	113	+0.7	9.6	99	+2.1
F.....	−2.1	16.0	115	+2.1	13.2	85	+4.2
G.....	−0.6	25.5	89	+0.9	15.6	64	+1.5
K.....	+2.7	17.9	283	+4.9	16.9	160	+2.2
M.....	+3.0	17.0	42	+7.5	17.6	38	+4.5

The galactic limits were arbitrarily assumed at $\pm 40°$, the limits used in earlier investigations. These are somewhat outside the real limits of the Milky Way and were used purposely in order to show as clearly as possible any differences between the two regions. This practice was considered preferable to the adoption of narrower limits on account of the relatively small number of stars. Such limits should at least not exaggerate any real differences. R is the mean algebraic residual for the group, and mean V, the average velocity, the signs being ignored.

The first thing to attract attention in these results is the algebraically smaller values of R for the galactic than for the non-galactic regions in all except the stars of class B, which exhibit a contrary condition.[1] This table also shows the markedly larger average velocities which exist in the galactic stars of the earlier classes, to which attention has been called by others. Our present interest in this table lies chiefly in the sizes of the discordances which are shown in the residual velocities of galactic as compared with non-galactic stars. If the term K is in reality a constant error it must from the nature of the case be independent of the region of the sky. The facts as derived from Table I seem to be that these residuals are different for the galactic and non-galactic regions, not only in the three classes which show large values for the K term, but also for the other three classes as well, that these differences are of considerable size, and that they appear to be systematic.

It may be noted that the algebraic mean of the galactic and non-galactic values of R are essentially the same as the K term for the three classes which have exhibited the latter. For the other three classes which showed no appreciable K term the means of the values of R are essentially zero.

In order to test whether there is any considerable constancy in smaller regions, the sky was arbitrarily divided into 72 regions, each 2^h in right ascension by $30°$ in declination, and the B stars of $3^{M}.0$ and fainter in each region were combined into one mean. The angular distance from the assumed solar apex was determined, using

[1] The same phenomena appear in the residuals of the B stars which were derived by eliminating the term K as well as the solar motion, (V_1).

the mean α and δ of the group. Although not strictly rigorous, this practice is amply accurate for such an investigation as this. Some of the regions were combined where there were a very few stars and they fell within a square 30° on the side or but little larger. A few single stars were omitted because of their isolation.

In order to eliminate to a large extent the effect of the larger velocities near the vertices of the ellipsoid, all stars within 40° of those vertices were omitted. There resulted 129 stars in 20 groups. Solutions were made for the solar velocity, preferential motion toward 7^h, $+64°$, and the K term. The results, together with two other solutions to be explained later, are given in Table II. The value of K derived from all of the 20 regions is in good agreement with that found from all of the B stars.

TABLE II

No. of Equations	No. of Stars	$V_\odot$	V_s	K
		km	km	km
20.....	129	−20.5	+0.8	+ 4.5
13.....	89	−19.8	+0.4	+ 0.8
7.....	40	−20.1	+0.1	+11.3

An examination of the residuals of the different groups, however, showed a marked tendency to divide themselves into two classes, those with positive signs and numerically averaging about as large as the K value and those of about the same size but with negative signs. This feature was so pronounced that separate solutions were made for the two groups, with results as shown in the second and third lines of Table II.

One-third of the regions (and less than one-third of the stars) have produced the positive residuals which give rise to the constant-error term, two-thirds of the regions showing only a very small excess of positive residuals.

As the value of the solar velocity is in good agreement for all of the solutions, we are forced to the conclusion that the differences are practically all in the excesses which give rise to the K term. An examination of the residuals from the third solution indicates that *three* different values of K (instead of two) would represent these

20 regions quite closely. The grouping of the stars used in this investigation by areas of considerable extent frees the foregoing values of K from criticism on the ground that they are simply accidental values. Individual stars would be open to such criticism, but it can scarcely hold for groups even as small as these when they represent areas of any considerable extent. It seems evident that there is a lack of anything like the uniformity which we should expect from a strictly constant error. The peculiar tendency to fall into two or three fairly well-defined values of K in these 20 regions is perhaps significant.

An examination of the numbers of the stars with large inherent velocities of approach and recession indicates a preponderance of positive velocities in classes B, K, and M and the contrary for classes A, F, and G. All of the stars of classes B, K, and M show a similar excess of positive velocities, equality for classes A and G, and slightly the contrary for class F. All of the 1300 stars show a decided excess of positive velocities. If the K term is in reality some form of constant error, its application tends to obliterate these peculiarities, so that the foregoing evidence is not wholly independent.

Table III contains the results of an examination of the residual velocities (R) and mean inherent radial velocities (V) of the stars within 40° of the apex and antapex of the sun's way after elimination of the solar motion, using the value of 19.5 km toward 18^h, $+30°$ and omitting K.

TABLE III

	Apex			Antapex		
	R	Mean V	No. Stars	R	Mean V	No. Stars
		km		km	km	
B.....	+ 2.2	8.1	21	+5.7	7.2	51
A.....	+ 2.1	9.1	28	−1.7	12.2	22
F.....	− 2.2	15.7	27	−1.4	14.9	27
G.....	− 6.4	16.9	24	−5.0	24.6	14
K.....	− 2.3	18.0	53	+8.3	16.9	66
M.....	−12.3	18.1	7	+6.2	20.4	12

Table IV contains the results of an examination of the *numbers* of stars (and their average inherent velocities), *moving toward* the

solar apex and antapex, i.e., positive velocities in the antapex and negative in the apex region are considered as moving *toward* the antapex and *vice versa* for the apex region.

TABLE IV

	Stars Moving Toward			
	Apex		Antapex	
	No. Stars	Mean V	No. Stars	Mean V
		km		km
B	22	6.7	50	7.8
A	23	13.5	27	7.9
F	26	15.5	27	15.7
G	22	15.1	16	26.0
K	47	14.8	72	20.4
M	4	26.4	15	17.7

The peculiar and possibly significant thing about these results is that in the three classes yielding large K terms (B, K, and M) we not only find an actual excess of stars in the antapex regions of all three classes but roughly the same number of stars *moving toward the antapex* as the number in that region. Further than this, we find that the ratios between the numbers of stars *moving toward the antapex* and in the opposite direction in these three classes are roughly proportional to the values of K obtained and more closely by the expression

$$K = 1 + \frac{\text{No. stars moving toward antapex}}{\text{No. stars moving toward apex}}.$$

It is further to be observed that in the regions discussed the stars of classes B, K, and M (those with large K terms) show generally large excesses of positive residuals toward the antapex region. Class A shows a considerable excess in the opposite sense and classes F and G very nearly balance. It seems premature to attempt a discussion of these points until the fundamental one of the nature of the K term is better understood. It may be pointed out, however, that such peculiarities as the above should be studied in connection with other evidence bearing on the nature of this term.

There appears to be no reason a priori to consider the residuals which we have found as being constant error any more than motion of some kind. While the physical explanation of pressure which has been offered has a firm basis, it remains to be shown that there is any such effect in the stars with large K terms, and, if so, any connection with the observed residuals. Neither do I see any reason to assume that there is a K term *superposed* upon peculiar or systematic motions. It is certainly most desirable to ascertain whether there are effects of pressure in any class of stars, but up to the present time I do not think that we are justified in assuming that the radial velocities in general are affected in such a way.

So far as the peculiar residuals which have been found in different portions of the sky are concerned, there appears no reason why they may not be some form of motion. We have no reason to assume that the motions of considerable groups of stars in opposite portions of the sky are the same. In fact, the accepted ideas of star-streaming, increase of velocity with increase of supposed age of the stars, and the magnitude-velocity equation would lead us to expect some such general differences of motion in widely separated portions of the sky.

Whether the differences found are large enough to disprove *constancy* sufficiently to require the abandonment of the term K as constant may be open to question. In my opinion the lack of uniformity in the residuals derived from galactic and non-galactic regions and from areas of considerable extent, taken in connection with other eccentricities of residuals which have been observed, is sufficient to show the necessity of proving a physical effect or a strictly constant error in the stars of classes B, K, and M before ascribing the residuals observed to other than motion.

Should it be confirmed that the peculiarities observed are in reality due to motion and not to a constant error, the results given in the tables are extremely suggestive. These results indicate:

1. That the non-galactic stars of all six classes are moving outward as a whole with respect to the instantaneous position of the sun, the velocities of the B, K, and M stars being greatest.

2. That the *galactic* stars of classes A, F, and G as a whole are approaching the observer with small velocities, while the galactic stars of classes B, K, and M are moving away.

3. That the outward movement of the galactic stars is in excess for the B stars, but that in all the other classes the greater outward movement is in the non-galactic stars.

4. That the peculiar preference of the stars of classes B, K, and M for the geńeral region of the antapex of solar motion, taken in connection with the proper motions of these classes, and peculiarities in the residual velocities in the regions of the apex and antapex, lead to the belief that they may be indicative of some far-reaching structural principle in the arrangement of these classes of stars.

What effect uncertainties of the solar motion either in amount or direction may have upon the above indications is not known. The interpretation of the phenomena with which we are dealing is a matter requiring great care and a due reserve is felt with respect to any explanations suggested.

CONCLUSIONS

1. After eliminating the solar motion from the stars of each spectral class the residuals are different for the galactic and non-galactic regions in all of them.

2. The algebraic mean of these residuals for galactic and non-galactic regions agrees reasonably well with the constant-error term (K) found for the same spectral classes in the solutions for the solar motion.

3. The differences between galactic and non-galactic residuals are of considerable size and appear to be systematic.

4. Other large differences of an apparently systematic nature have been found in large areas of the sky, which add to the suspicion of irregularity in the tendency of motions in different regions.

5. The results of this investigation indicate a lack of the uniformity in the residuals giving rise to the term K which we should expect from a strictly constant error or physical effect, and until such effects are proved to be present in the observed velocities, the doubt exists that the peculiarities observed are other than motions.

OBSERVATORIO NACIONAL ARGENTINO, CÓRDOBA
October 4, 1915

NOTE ADDED BY AUTHOR, UNDER DATE DECEMBER 6, 1915

Subsequent investigations have disclosed large and apparently systematic differences between the values of the K term as derived from the northern and southern stars separately. The values obtained and the corresponding values of $V_{\odot}$ are given below:

	$V_{\odot}$		K	
	North	South	North	South
	km	km	km	km
All spectral classes 2.9 and brighter	−10.8	−21.4	−5.7	+3.4
B 3.0 and fainter	−24.8	−20.6	+9.6	+5.6
A 3.0 and fainter	−23.0	−19.7	+3.8	+2.9
A 3.0 and fainter, omitting ellipsoid vertices	−16.6	−22.2	0.0	+8.3
A 3.0 and fainter, omitting ellipsoid vertices and 2 large values	−17.2	−16.0	−0.9	+5.4
F and G 3.0 and fainter, galactic	−22.9	−14.5	+1.1	−4.3

A MORSE OPTICAL PYROMETER ADAPTED TO A WIDE RANGE OF LABORATORY USES

By W. E. FORSYTHE

The following is a description of an optical pyrometer of the Morse type (generally known as the Holborn-Kurlbaum pyrometer) that has been constructed for use in Nela Research Laboratory. It is thought to have some advantages over existing forms. A photograph of the complete instrument is shown in Fig. 1. In Fig. 2 is shown diagrammatically the arrangement of the different parts.

The objective lens *A* is an achromatic lens 3.5 cm in diameter. If the pyrometer were to be used only with a particular so-called monochromatic filter (e.g., red glass) before the eyepiece, there would be no need of having anything but simple lenses. However, it is quite often desirable to use different filters before the eyepiece, in which case quite a change in focus would be required if only simple lenses were used.

As may be seen from the diagram, the tube holding the objective lens is not a part of the tube to which the rack and pinion are attached. The reason for making it a separate tube was to increase the range of magnification. For the same reason this tube is made in two parts, there being a joint at the point marked *B*. Thus with only a comparatively short rack for focusing, adjustments from a magnification of little more than two times down to a focus for parallel light can be obtained. This is accomplished by moving the tube holding the objective lens backward or forward in the holder *C*. The tube can also be taken apart at the point *B* and only the short piece containing the objective lens used. The lens can also be reversed in this short tube and the tube reversed in the holder *C*. Thus the lens can be placed very near the diaphragm *D*. At this position the pyrometer is focused for parallel light. This feature has been found quite desirable. The pyrometer

may be calibrated by comparison with a black body where large magnification cannot be used conveniently, and the pyrometer may also be used to measure the brightness of either distant objects or, by a slight change, to measure the brightness of a lamp filament which is only about 0.1 mm in diameter.

The pyrometer lamp K has a tungsten filament somewhat of the shape shown in Fig. 3. The pointer P and the small bend in the filament indicate the point of the filament under observation. The filaments used are of three sizes (0.1 mm, 0.061 mm, and

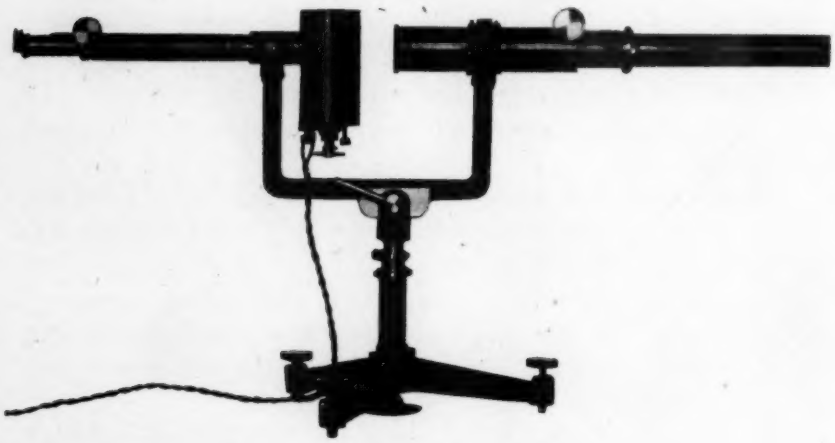


FIG. 1.—Optical Pyrometer

0.037 mm in diameter), but for the most part the second size is used, as with this the disappearance is satisfactory. The length of these filaments is approximately shown in Fig. 3. The (0.061 mm) filament requires about 0.46 amperes at about 3.3 volts to apparently match in brightness the black body at the temperature of melting palladium. The pyrometer lamp bulb is about 5 cm in diameter. These bulbs were selected with very great care in order to obtain bulbs as free from defects as possible.

The pyrometer lamp is so mounted that it may be moved across the field of view by a screw motion. Thus the particular part of the filament that it is desired to use may be easily brought into the center of the field of view. In addition to this the mounting is so

arranged that all other possible adjustments, such as raising and lowering, turning and tipping the bulb, can be easily obtained.

As it is sometimes very desirable to have the pyrometer filament horizontal and at other times vertical, the pyrometer lamp-holder *F* and the eyepiece telescope were made so as to rotate in the collar *G*. By this means the pyrometer filament can be set at any angle. This has been found very convenient when measuring the temperature of a lamp filament. Owing to the variation from the cosine law of emission,[1] it is desirable to have the pyrometer filament

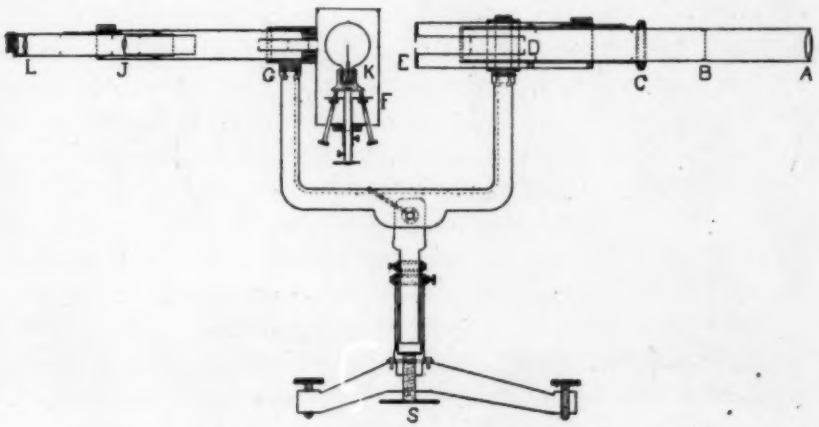


FIG. 2.—Optical Pyrometer

parallel to the background filament. In this case it is the brightness of the central part of the filament that is measured rather than the mean brightness across the filament. This average brightness has been shown to be about 3 per cent greater than the normal brightness.

It is sometimes desirable to use two different pyrometer lamps in the same optical pyrometer. This is necessary when two different observers have to use the same pyrometer but desire to use different pyrometer lamps. It is also desirable to have a second pyrometer lamp to use occasionally to check the calibration of the pyrometer lamp that is generally used. To be sure, one

[1] *Astrophysical Journal*, 36, 345, 1912.

pyrometer lamp can be removed and the other put in its place, but a much more accurate procedure is to have the holder large enough so that both lamps can be kept in the holder at all times. The holder for the pyrometer lamps can be brought exactly into place. This can be accomplished by turning the holder or by sliding it along guides. If the lamps are kept in the holder, they will be more likely to be used in the exact position in which they were calibrated, which is very important for accurate work.

The pyrometer lamps with tungsten filaments have a very long life as they are used in this kind of work. They are seldom operated at a brightness above that necessary to match that of a black body at the temperature of melting palladium (1822° K). A particular lamp that has been in use for about two years, and has during the greater part of that time been used almost every day, shows an exceptionally good performance. When it was first calibrated two years ago it required 0.4573 ampere to apparently match the black body at the temperature of melting palladium. A recent calibration showed that it now required 0.4578 ampere to apparently match the black body at the same temperature. As this lamp has been operated for about 300 hours in all, it is seen to be very satisfactory. This pyrometer lamp has a 0.062 mm filament.

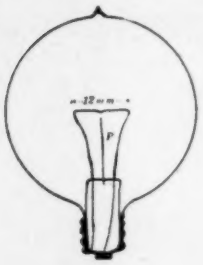


FIG. 3

As a ready means of checking the calibration of the pyrometer lamp a large filament (0.25 mm) tungsten lamp, standardized by comparison with the black body by the substitution method, is used. By using this lamp as a secondary standard the necessity of setting up and calibrating a standard black body can often be avoided. The calibration of the pyrometer lamp can be checked much more readily by means of this secondary standard than with the black body.

The opening at *E* was made for two purposes: (1) to allow the lamp-holder and eyepiece telescope to revolve, as was pointed out above; and (2) to permit the insertion of a rotating sector, since it has been shown[1] that if a rotating sector disk is used to reduce the

[1] *Astrophysical Journal*, 42, 303, 1915.

intensity of the light incident upon the pyrometer lamp it should be placed as near the pyrometer lamp as possible. Though for the most part it is much better to use rotating sectors to reduce the intensity of the incident radiation, owing to the difficulty of securing satisfactory filters of neutral tint, it is sometimes necessary to use such filters. This is especially true when using light of the shorter wave-lengths, as the ratio of change of brightness for a given change in temperature is much greater for shorter wave-lengths. A holder for the absorbing glasses is located at *E*.

The telescope, consisting of the objective lens *J* and the eyepiece *L*, is used to observe the pyrometer filament and the projected background. This telescope is so constructed that the magnification can be altered. Thus it is possible to use the greatest magnification allowable and still have a good disappearance of the pyrometer filament against the background. One objection that has been raised against tungsten pyrometer filaments is that they disappear against the background too easily. This is due for the most part to the optical system used to observe the pyrometer filament. With an optical system of good resolving power and large magnification it is often impossible to get an apparent disappearance of the pyrometer filament against the background. It has been pointed out[1] that in order to obtain a disappearance it is sometimes necessary to use an eyepiece with less resolving power than would otherwise be convenient. It is not to be understood that the trouble with disappearance is limited to tungsten pyrometer filaments. With carbon pyrometer filaments it is often difficult to obtain a disappearance. Tungsten and carbon pyrometer filaments of the same size behave differently, owing to their difference in deviation from Lambert's cosine law.[2]

At *D* is located the diaphragm that has been shown[3] to be necessary in order to avoid an error due to diffraction at the pyrometer filament. This error is caused by changing the solid angle of the radiation incident upon the pyrometer filament, due to changing the position of the objective lens. As the solid angle is changed,

[1] *Physical Review* (N.S.), **4**, 163, 1914.

[2] *Astrophysical Journal*, **36**, 345, 1912.

[3] *Physical Review* (N.S.), **4**, 163, 1914.

the diffraction around the filament will be changed. If, for example, this solid angle is increased, more light will be diffracted around the filament. The filament will thus require less current to apparently match in brightness a particular background. The size of this diaphragm is so chosen that it is completely filled by the cone of light from the objective lens even when the lens is moved to the farthest distance possible from the pyrometer filament.

The real limiting diaphragm is located at *H*. The upper limit to the size of this diaphragm must be so chosen that it is smaller than the cross-section of the cone of rays limited by the diaphragm *D*. A limiting diaphragm is necessary, as the apparent brightness of any radiating body depends upon the solid angle through which the radiation passes before it enters the eye. Thus, with a limiting diaphragm, the brightness of any radiating solid, as observed with a telescope, is independent of its distance so long as the limiting diaphragm is filled. This same argument applies to the pyrometer filament, and the two should always have the same limiting cone. What really determines the size of this diaphragm is the resolving power of the eyepiece telescope. The size of this diaphragm must be so chosen that with the magnification necessary to use, disappearance of the pyrometer filament can be obtained. The foregoing diaphragms *D* and *H* were respectively 1 cm and $2\frac{1}{2}$ mm. The other diaphragms, shown in the eyepiece telescope and the objective telescope, are for the purpose of lessening the stray light. The support for the pyrometer, as shown in Fig. 2, is so constructed that the instrument can be rotated, raised or lowered, or set at any angle with the horizontal. The pyrometer can also be raised a small amount by means of the screw *S*.

This pyrometer has been used in connection with the less portable forms of the Holborn-Kurlbaum optical pyrometer previously described.[1] The accuracy obtained with this pyrometer is about the same as with the larger forms.

It is interesting to note that the principle of this pyrometer (generally referred to as the Holborn-Kurlbaum pyrometer), patented by Morse in this country about 1900, was discovered and used back in 1888 in one of the lamp factories. At that time a

[1] *Physical Review* (N.S.), 4, 163, 1914.

certain furnace was operated at such a temperature that it matched the standard lamp at about 80 volts.

The author does not desire to claim originality for the above-described instrument. The pyrometer as constructed is the outgrowth of several years of use of such instruments both in this laboratory and in the University of Wisconsin. Different members of this laboratory as well as others elsewhere have given many valuable suggestions.

NELA RESEARCH LABORATORY
NATIONAL LAMP WORKS OF GENERAL ELECTRIC COMPANY
NELA PARK, CLEVELAND, OHIO
April 1, 1916

A NOTE ON SPECTROPHOTOGRAPHY

By M. LUCKIESH

Two ideal photographic emulsions that would meet with high favor in spectrophotography are, respectively, one that is equally sensitive to radiant energy of all wave-lengths, and another that is sensitive to radiant energy of various wave-lengths in quite the same relative proportions as the average eye. It is well known that no such ideal emulsions are available. In fact, no emulsions are available which even approximately satisfy the foregoing conditions excepting for relatively short spectral ranges. It is a common procedure to resort to colored filters for the purpose of obtaining the same final results as would be obtained by the ideal emulsions. Obviously the functions of the colored filters is to suppress, in proper proportions, energy of various wave-lengths to which the emulsion is more sensitive, thus "evening up" the photographic action in the emulsion throughout the desired spectral range, or accomplishing other desired results in the same manner. It is not difficult to make colored filters which roughly compensate for the lack of uniform spectral sensibility of the emulsion, but it is usually a long and tedious process to make a filter for a given photographic emulsion which accurately accomplishes the same final result as either of the ideal emulsions mentioned above. Further complications sometimes arise in spectrophotography owing to the lack of uniform spectral distribution of energy in the illuminant used in obtaining spectrograms of transmitted and reflected light, and also owing to the effect of prismatic dispersion in spectrographs in which prisms are employed. Templates may be used instead of colored filters, and for accurate work are perhaps as easily made as colored filters.

The method described here has been employed by the writer on various occasions during the past few years with considerable success and satisfaction. The procedure is so simple that it is probable that it has been used by others, but, inasmuch as the writer is not aware that this is a fact, it appears worthy of a brief descrip-

tion. The method consists of making a spectrophotographic filter through which spectra are photographed. The method will be discussed first for a grating spectrograph. A photographic plate which is to be employed for spectrophotography is placed in its ordinary position in the spectrograph and the spectrum of a light-source yielding a continuous spectrum, such as the tungsten lamp, is photographed. Several spectrograms obtained by different exposures are photographed upon the plate. The latter is

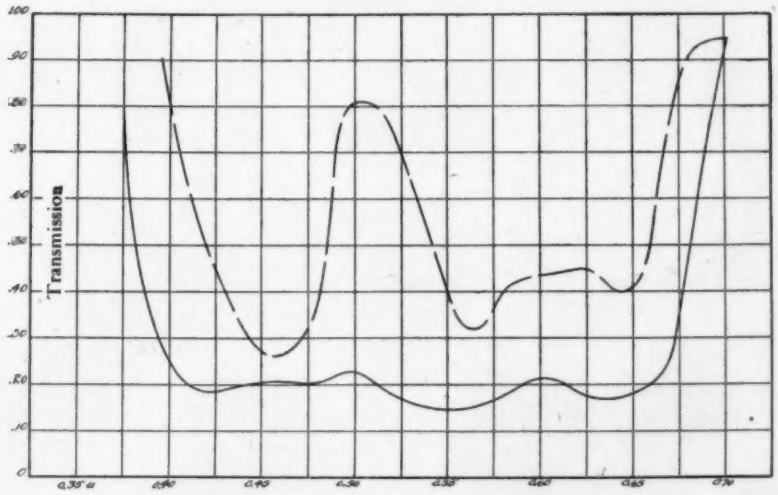


FIG. 1.—Broken line: Transmission of spectrographic filter. Full line: Transmission of a continuous spectrum through the filter, with Cramer Spectrum plate and grating spectrograph.

developed, fixed, dried, and replaced in its original position in the plate-holder. Another plate of the same kind is placed on top of the first plate and spectra are now photographed upon the second plate through the negatives on the first plate. The photographs should be made on the first plate with the sensitive side reversed from its normal position. This makes it possible to place the sensitive side of the second plate in contact with that of the first plate, thus avoiding the errors that might arise in refocusing the instrument, as would be necessary if the sensitive films in both cases were not in the same plane. This method compensates fairly well for the non-uniformity in the spectral sensibility of the emulsion

and for the non-uniform spectral distribution of energy in the illuminant. There is a best relation between the exposures on the first plate and the uniformity of the photographic effect on the second plate throughout the range of wave-lengths to which the plate is sensitive. After ascertaining the best density for the "spectrophotographic filter," the writer uniformly exposes the first plate completely across its width by moving the plate-holder at a uniform and proper speed across the opening. After develop-

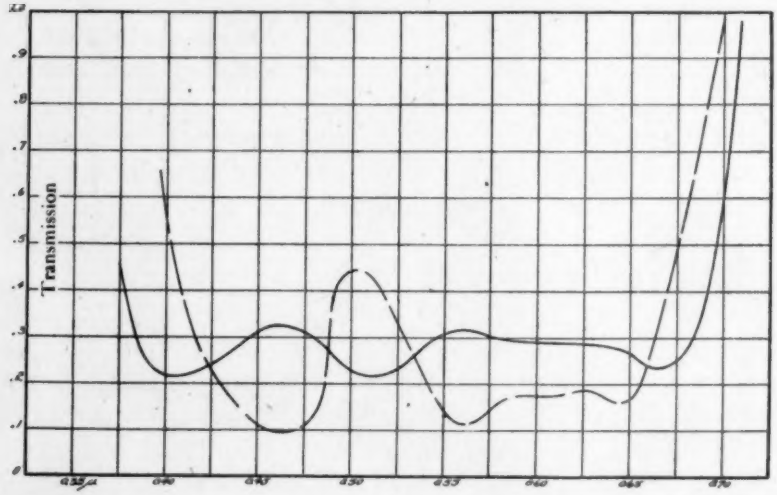


FIG. 2.—Same as Fig. 1, except that longer exposures were given. "Filter" was more dense.

ing, fixing, and drying, the wave-length scale is ruled upon the "filter," providing that it has not been photographed during the original exposure. The result obtained by this simple procedure closely approximates that which would be obtained by means of an ideal emulsion of uniform spectral sensibility and an illuminant of uniform spectral energy-distribution.

To illustrate the results, actual spectrograms might be used, but a much better idea of the compensatory effects of the spectrophotographic filters is obtained by means of photometric data obtained by measuring the transmission of the "filters" and of the final spectrograms at different wave-lengths. The data might ordinarily be presented in terms of density or opacity, the former

being the logarithm of the opacity and the latter the reciprocal of the transmission coefficient, but, inasmuch as the eye usually estimates transmission coefficients in judging photographic transparencies, the spectral transmission curves of the "filters" and the final spectrograms are presented.

In Fig. 1 the broken line represents the spectral transmission curve of the spectrogram of incandescent tungsten light (the lamp operating at about 20 lumens per watt) on a Cramer Spectrum

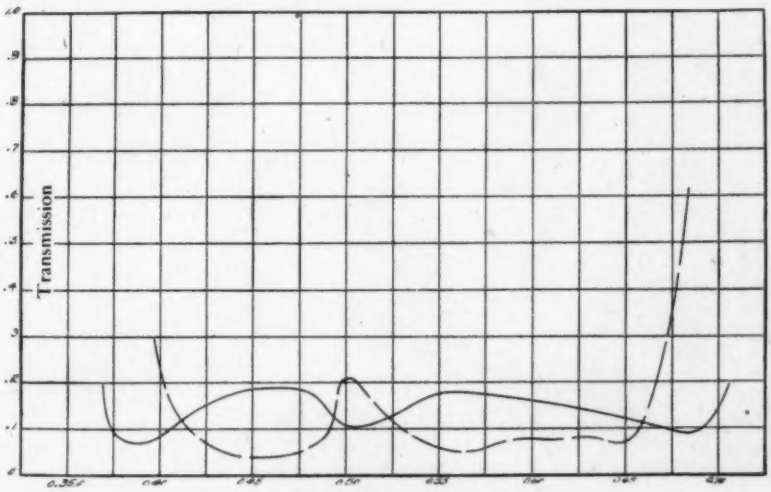


FIG. 3.—Same as Fig. 2, except that the filter was still more dense

plate. The negative was of rather light density. The full line represents the spectral transmission-curve of the spectrogram of the same illuminant taken through the first negative with the latter in contact with the second plate and in its proper position according to the wave-length scale. It is seen that throughout most of the region of sensitivity for ordinary exposures, from 0.4 μ to 0.67 μ, the photographic effect is nearly constant and therefore this represents a satisfactory correcting screen. The absorption of the glass, emulsion, and any slight fog has been eliminated in the measurement.

In Fig. 2 the broken line represents the spectral transmission curve of a spectrogram of the illuminant on a Cramer Spectrum

plate, the negative in this case being more dense than in the previous case. The full line shows the effect on the spectrogram of the same illuminant taken through the first negative and indicates an overcorrection; that is, the maxima and minima of the full-line curve correspond respectively to the minima and maxima of the broken-line curve.

In Fig. 3 the two curves represent data corresponding to those shown in the previous cases, but the exposures were greater. The overcorrection is again evident.

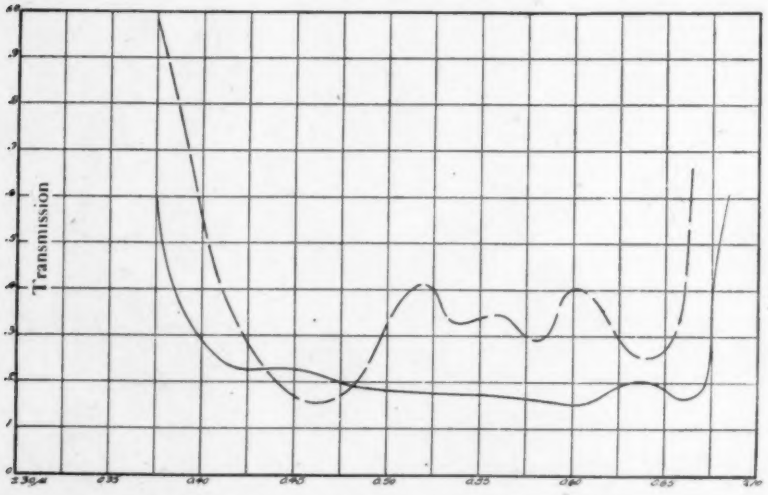


FIG. 4.—Similar results with a Wratten & Wainwright Panchromatic plate

In Fig. 4 are shown similar results with a Wratten & Wainwright Panchromatic plate. The spectrophotographic correcting-filter in this case is quite as satisfactory as that shown in Fig. 1.

It is thus seen that with a grating spectrograph the compensation for the non-uniform spectral energy-distribution in the illuminant and for the non-uniform spectral sensibility of the photographic emulsion is satisfactory. For qualitative studies this method has proved highly satisfactory by practically eliminating any misinterpretation of the spectrograms which often arises owing to the insensitive spectral regions encountered with photographic

plates. For quantitative measurements the method is quite helpful because, with a proper spectrophotographic filter, all densities throughout a large spectral range fall on the straight portion of the intensity-density curves. In much work the slopes of the intensity-density curves for energy of different wave-lengths may be assumed equal.

The results obtained with a prism spectrograph are shown in the remaining illustrations. In this case the variable dispersion of the prism is often annoying, owing to the extreme diminution

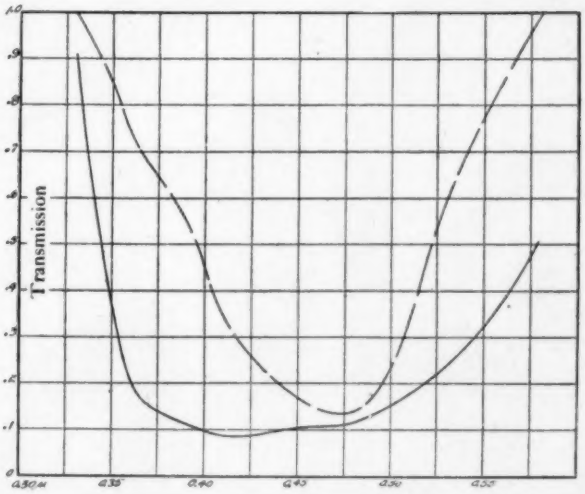


FIG. 5.—Similar results with a glass prism spectrograph and Seed 26 plates

of energy-density in the short-wave region of the spectrum. In Fig. 5 the broken line shows the spectral transmission curve of a spectrogram of the same illuminant on a Seed 26 plate. The resulting spectrogram through this spectrophotographic filter is shown by the full line. This plate is not very sensitive to energy of longer wave-lengths than 0.50 μ, and the glass prism begins to absorb energy quite appreciably at 0.35 μ, becoming practically opaque at 0.30 μ. This is an ordinary plate and therefore its spectral range is considerably shorter than the panchromatic plates. However, it is seen that even in a prism spectrograph the compensating method which has been described is quite effective

throughout the region of chief sensibility of the photographic emulsion.

In Fig. 6 similar results are shown for greater exposures. The correction is perhaps slightly better than in the preceding case.

The method has been used successfully with various kinds of photographic plates and with prism and grating spectrographs. Promising results have been obtained with a quartz spectrograph as far as 0.30 μ, using a source giving a continuous spectrum. In a great deal of spectrophotographic work, such as the examination

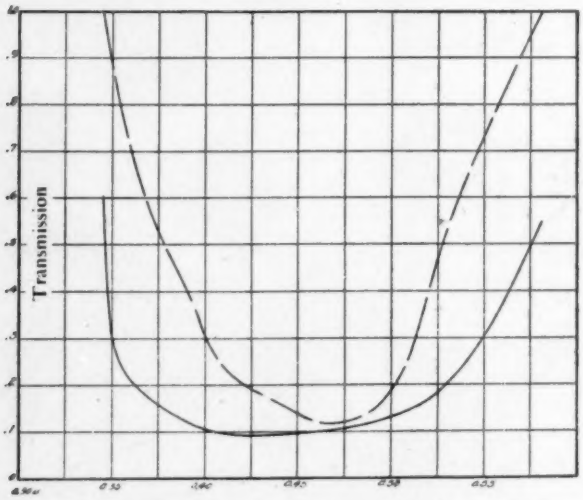


FIG. 6.—Similar to Fig. 5, with a more dense "filter"

of glasses for protecting the eyes, the region of most importance is the near ultra-violet, 0.29 μ to 0.40 μ, which is transmitted by glass. Here a method which compensates as does the foregoing is quite desirable.

SUMMARY

A simple method has been described for making a spectrophotographic filter to be used with a grating instrument for obtaining practically the same results as would be obtained in spectrum photography with an ideal photographic emulsion of uniform sensibility throughout its chief range of sensitiveness and with an illuminant emitting equal amounts of energy of all wave-lengths to which

the emulsion is chiefly sensitive. The method is equally applicable to prism instruments, in which case the effect of prismatic dispersion is largely compensated, as well as the non-uniform sensibility of the emulsion and the non-uniform spectral distribution of energy in the illuminant.

The method has been used with both prism and grating spectrographs and the details and some results are presented.

NELA RESEARCH LABORATORY
NATIONAL LAMP WORKS OF GENERAL ELECTRIC COMPANY
NELA PARK, CLEVELAND, OHIO
March 24, 1916

MONOCHROMATIC PHOTOGRAPHY OF JUPITER AND SATURN[1]

By R. W. WOOD

In the *Astrophysical Journal*, **36**, 75, 1912 I gave an account of the results obtained by photographing the moon through ray-filters transmitting limited regions of the spectrum. Marked differences in the distribution of light and shade were observed in the different pictures, the most conspicuous being an extensive dark deposit around the crater Aristarchus, which appeared in the picture made by means of ultra-violet light but was quite absent in the one made by the yellow rays. Laboratory experiments on the appearance of various rocks and minerals, when photographed through these same ray-filters, made it appear probable that the dark spot was due to sulphur or some sulphur-bearing rock.

It seemed quite likely that results of even greater interest would be obtained if the method of monochromatic photography were applied in the case of planets, but instrumental facilities were not at my disposal at the time.

These earlier experiments were made with a 16-inch nickeled glass mirror of 26 feet focus, mounted on the equatorial of the Princeton Observatory, which unfortunately was not adapted to the very accurate guiding necessary in work of this nature. As it appeared probable that increased definition and magnification would bring out regions of smaller area showing selective reflection, and as there was every reason to suppose that the planets would show peculiarities worthy of study, I determined to repeat and improve upon the earlier experiments. In this work I have been aided by a grant of $200 from the Gould Fund of the National Academy. The preliminary experiments were made at East Hampton, during the past summer.

The Naval Observatory placed at my disposal a very good Gaertner coelostat, which was used in conjunction with a horizontal reflecting telescope of 56 feet focal length and 16 inches aperture,

[1] *Contributions from the Mount Wilson Solar Observatory.* No. 113.

figured by Mellish and nickeled by the electrolysis of a dilute solution of nickel fluoborate as described in the *Astrophysical Journal* for November 1915 (**42**, 365). The nickel deposit was necessary, since silver reflects only about 4 per cent of the light in the ultra-violet region of the spectrum utilized in the work.

A new mirror cell was made for the coelostat, large enough to carry a 16-inch flat, which was also nickeled, and the instrument mounted on a very solid pier of brick and cement, sheltered by a small house with a hinged roof. A double-slide carrier for the plate-holder and ray-filters, with two slow motions and two eyepieces for following, was made of sheet brass, and mounted on a heavy pedestal of iron.

The photographs made with this apparatus, while distinctly better than those made with the 26-foot mirror, failed to show the very fine details which were desired. This was due partly to bad seeing and partly to the difficulty of following a rather rapid periodic oscillation or drift of the image originating in some small error or errors in the gear wheels which could not be located or remedied.

On this account it appeared best to study the behavior of the various ray-filters, and the spectrum range of the light after reflection from two or more surfaces of silver or nickel and transmission through the filters, rather than to waste time in trying to secure a more accurate drive of the coelostat. These experiments showed that it would be feasible to complete the work with a large reflecting telescope with silvered mirrors, using as an ultra-violet screen a cell filled with dense bromine vapor. Such a cell transmits the entire ultra-violet region below λ 3500, while silver reflects the region λ 3300–3500 sufficiently well to make photography possible in this region with silver-on-glass mirrors.

Photographs were made of the moon and Jupiter with infra-red, yellow, violet, and ultra-violet light, and the times of exposure, with the screen, were determined for both silvered and nickeled mirrors. As a result of this preliminary work, I was able to get satisfactory results at once with the 60-inch reflector of the Mount Wilson Observatory, which was placed at my disposal for four nights in the latter part of October.

With this instrument a very complete set of photographs was made of the moon, Jupiter, and Saturn, by light of the four regions

of the spectrum referred to above. Results of very great interest were obtained in the case of the two planets, for dark belts were shown by violet and ultra-violet light of which no trace appeared visually or on the plates made by yellow light, while the infra-red pictures showed scarcely any trace of the belts ordinarily seen. By applying the methods of three-color printing to these plates we can produce a colored positive, in which the belts of selectively absorbing gases appear strongly colored; in other words, we produce an effect somewhat similar to what we should see if our eyes were sensitive to a wider spectral range.

In the earlier experiments, made several years ago, I used for the ultra-violet ray-filter a rather thick deposit of metallic silver on a thin plate of quartz or uviol glass. Silver films are, however, extremely opaque to the selectively transmitted region, which lies between λ 3000 and λ 3200, so much so that an exposure of two or three minutes is required for the full moon. With bromine vapor I found, however, that a fully exposed plate could be obtained in eight or ten seconds, and that if we use in addition a thin cell filled with a very dilute solution of potassium chromate, we obtain a transmitted region practically the same as with a silver film, but of about tenfold intensity. The silver film was accordingly abandoned in the present work and the bromine cell used exclusively in its place.

The cell was made by cutting a section from a square glass bottle and grinding the edges flat. A small hole was then drilled through the wall for the introduction of the bromine. The uviol plates (1 mm in thickness) were cemented with beeswax, which was found by Ribaud, in the course of his investigations on the absorption spectrum of bromine, to withstand the action of the vapor fairly well. A piece of asbestos paper, impregnated with a saturated solution of calcium chloride and strongly heated to drive off all of the water, was placed in the cell before the plates were cemented in position. This was necessary owing to the dampness at East Hampton, for, even when the bromine was thoroughly dried, moisture deposited on the inner surface of the uviol plates when the cell cooled off at night. After several hours there appeared also a deposit which resembled a light dew but which did not dry off when the plates were removed. It appeared to be of a greasy nature, and I attributed it to some volatile compound

formed by the action of the bromine on the wax. I subsequently found it on clean glass plates which had been standing on my desk for several days, and was quite at a loss for an explanation until Dr. Mees, of the Eastman research laboratory, called my attention to the fact that beeswax sublimes quite rapidly, a circumstance which I had not suspected, as it is one of the constituents of a supposedly reliable vacuum cement. The trouble was minimized by applying the wax to the outside of the cell only, after the glass plates were placed against the ends. The wax was rolled into thin cylinders between the fingers, and melted against the seam with a hot glass rod 2 mm in diameter. Cells prepared in this way showed no trace of the deposit for fifteen or twenty hours, and, as they were cleaned every evening and freshly filled, no further trouble was experienced from this source. Four or five drops of liquid bromine were introduced with a capillary pipette, and after vaporization was complete and most of the air expelled, which required a minute or two, the hole was closed with a plug of wax, touched with a hot glass rod.

The cell was 5 cm in thickness and was clamped in position in front of the plate-holder by means of two brass set-screws. Exposure with the quartz spectrograph showed that such a cell was transparent from λ 3500 to the end of the solar spectrum (λ = 2900), while it cut off completely all of the less refrangible rays capable of affecting an ordinary photographic plate.

If a more restricted region of the spectrum is required we may add a thin cell, say a millimeter or two thick, containing a very dilute solution of chromate of potash; the yellow color should be barely perceptible by daylight. The correct dilution can be determined by trial only, the quickest way being to use sunlight filtered through the bromine cell, a quartz spectrograph, and a plate of uranium glass. If the solution is not extremely dilute, it removes all of the ultra-violet. If just right, it removes the region λ 3250–3500, the combination transmitting the same region as a thick silver film, but with a much greater intensity (about tenfold). If the chromate-of-potash cell is used, it is necessary to give three or four times the exposure required with the bromine cell alone.

The infra-red screen transmitted the region above λ 7000, the yellow screen everything above λ 5000, and the violet the region

between λ 4000 and λ 4500. These screens were prepared from stained films of gelatine made by the Eastman company, and mounted in balsam between thin sheets of plate glass. They were of such a size that they could be substituted for the thick bromine cell, and were held in place by the same set-screws.

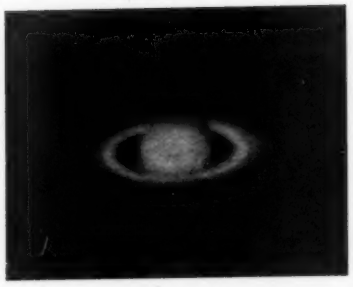
The work at Mount Wilson was commenced on October 22. The double-slide carrier with its attachments for the bromine cell was adapted to the 60-inch telescope, so that the work could be taken up without the construction of any new apparatus. The early hours of the evening were spent in making plates of the moon, which was full at the time, through the four ray-filters, while the rest of the night was given over to the planets.

The telescope was used at the 80-foot focus, and, as the mirrors were silvered, the region of the spectrum utilized for the ultra-violet photography was slightly less refrangible than in the earlier work with the silver film. Preliminary experiments had been made at East Hampton with a quartz spectrograph, on the spectrum of sunlight after three reflections from silver and transmission through the bromine cell (the conditions which obtained with the 60-inch telescope).

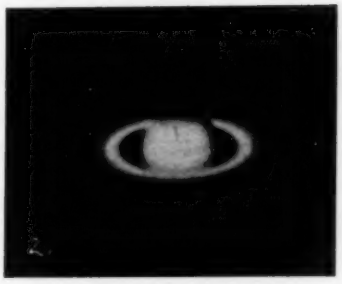
The most interesting results were obtained in the case of Saturn. The picture taken through the infra-red screen showed the ball of the planet practically devoid of surface markings, there being only the merest trace of the belts ordinarily seen. Through the yellow screen the planet presented its usual visual appearance, the narrow belts showing distinctly. On the plates made with the violet ray-filter (transmission λ 4000–λ 4500) a very broad dark belt surrounded the planet's equator, occupying the region of the planet which was brightest in yellow light. In addition to this dark equatorial belt, a dark polar cap of considerable size appeared in the pictures. So different were the two pictures that, were it not for the ring, it would have been difficult to believe that they represented the same object. In ultra-violet light the appearance was much the same, but the dark belt was not quite as wide, the bright region between the polar cap and the belt being distinctly broader. Photographs made with the four monochromatic filters are reproduced in Plate IV.

Two hypotheses suggest themselves in explanation of the dark belt. We may be dealing with a fine mist or dust which forms an

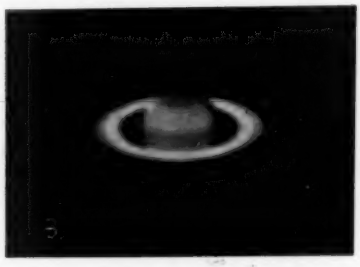
PLATE IV



1 Infra-red



2 Yellow



3 Violet

4 Ultra-violet

MONOCHROMATIC PHOTOGRAPHS OF SATURN

extension of the crape ring down to the ball of the planet. This hypothesis appears to be favored by the circumstance that, on the negatives made by the violet and ultra-violet light, the sky between the ball of the planet and the ring is distinctly denser than the region just outside. This would indicate that the region inside of the ring is filled with some material which reflects the short wave-lengths to a slight degree. No trace of this darkening appears on any of the plates made with the yellow screen, even on one that was many times overexposed, which appears to show that the phenomenon is real. This peculiarity was, however, not detected until the work was finished, and I should prefer to verify it or have it verified before recording it as an established fact. The luminosity is much too feeble to show in the prints.

Measurements of the density of the plates between the ring and the ball of the planet and the sky just outside of the ring have been made for me by Mr. Harold D. Babcock, of the Mount Wilson staff, with the Hartmann photometer. His results showed greater density between the ring and ball on both plates (yellow and ultra-violet, but the density was greater in the latter). His values for the densities are as follows:

Yellow Negative		Ultra-Violet Negative	
Sky outside of ring	1.1	Sky outside of ring	1.05
Sky between ring and ball	1.17	Sky between ring and ball	1.27

One's first thought is that slight darkening of the region between the ring and ball may be the result of halation. It appears to me, however, that if this were the case we should expect the effect to be more marked with yellow light, which penetrates the film and the glass plate and is reflected back, while the ultra-violet light does not even get through the photographic film, as is shown by the circumstance that the image does not appear on the reverse side of the film with prolonged development, as is the case with ordinary light.

I do not, however, believe that the dark belt is in reality due to the absorption of a dust ring, for measurements made by Mr. Ellerman showed that the belt extended higher up on the ball of the planet than the line of intersection of the plane of the rings. Moreover, it seems highly probable that the belt and the dark polar cap are to be explained in the same way.

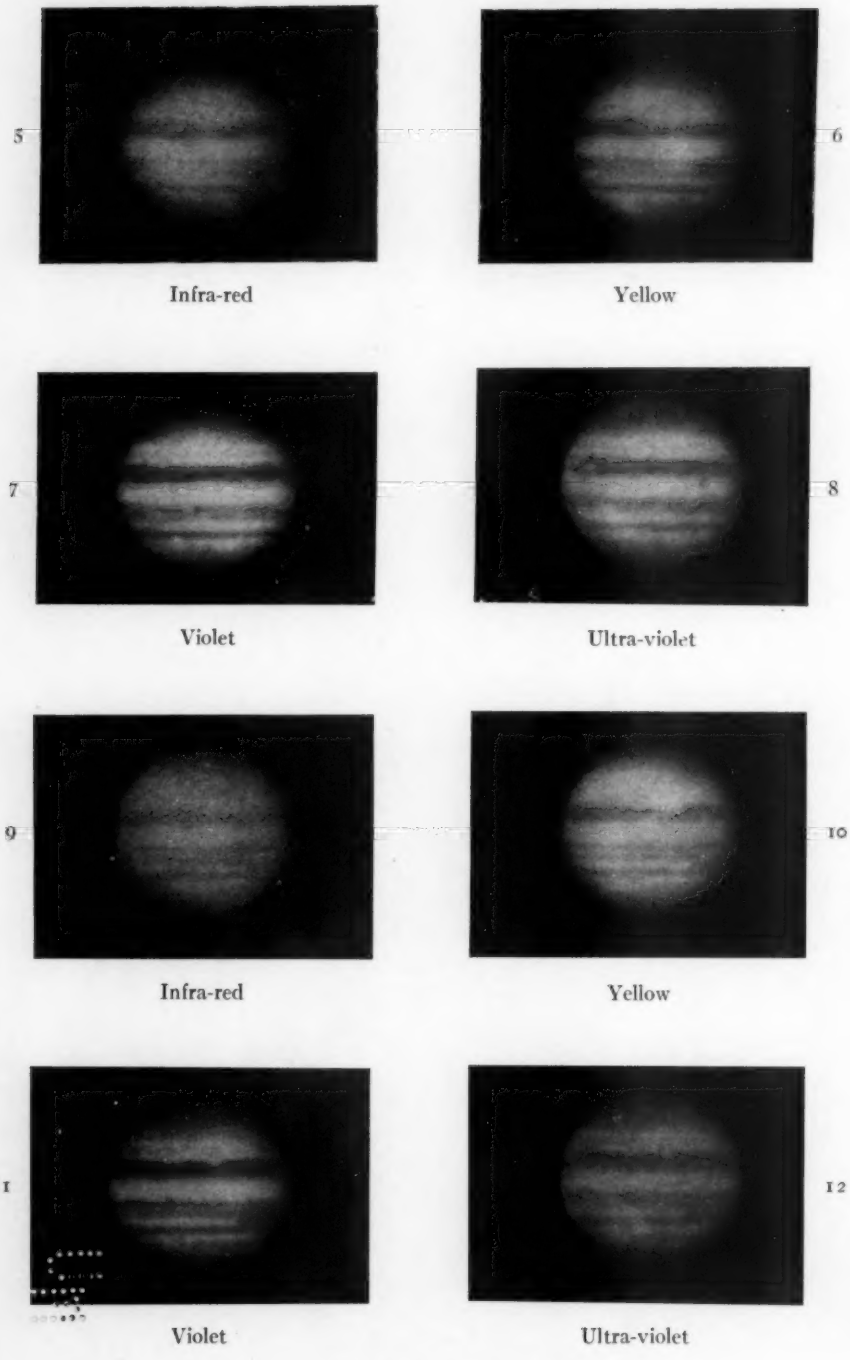
The second hypothesis assumes the existence in the planet's atmosphere of some substance capable of absorbing violet and ultra-violet light. This material might be a fine mist or dust, or some gas capable of absorbing the more refrangible part of the spectrum. Such a gas would be of a pale yellow color, and sulphur vapor and chlorine naturally occur to us. I have, however, examined the absorption spectrum of both of these gases in quartz bulbs, with the result that the absorption appears to be much stronger in the ultra-violet than in the violet, which is in disagreement with the circumstance that the band appears wider in violet than in ultra-violet light.

Two spectrograms of the planet have been made by Adams with the 60-inch reflector. In one, the slit of the instrument was parallel to the major axis of the ring and cut through the dark belt shown on the violet and ultra-violet photographs. In the other, the slit cut across the bright region above the belt. The spectra embraced the region between λ 5000 and λ 4000, and we should expect that between λ 4000 and λ 4500 the spectrum of the area occupied by the dark belt would be much darker (lighter in the negative) than the other. No difference could be detected, however, between the two negatives, except at the very end (λ 4000), where there was a very slight trace of the expected effect.

Of course it would be preferable to place the slit parallel to the short axis of the ellipse, and show the spectra of the two regions juxtaposed on the same plate, but the spectroscope available at the time did not permit of rotation. A quartz spectrograph is being planned for a more complete study of the question. The spectra were made in February 1916, and it is of course possible, though hardly probable, that the ultra-violet belt had in the meantime disappeared.

Another point of interest is the decrease in contrast between the inner and outer ring as the wave-length is decreased. This suggests that the outer ring contains so much finely divided matter that it shines in part by diffusion. On the infra-red photograph the ball of the planet is much brighter in comparison to the brightest part of the ring than on the violet and ultra-violet pictures. This again suggests a mist or dust in the planet's atmosphere which scatters the shorter wave-lengths. My infra-red photographs of landscapes

PLATE V



MONOCHROMATIC PHOTOGRAPHS OF JUPITER

have clearly demonstrated that we can obtain clear photographs through a blue haze by means of the spectrum region above λ 7200. In these photographs it will be remembered that the blue sky comes out black and the grass and foliage show white. (See *Physical Optics*, 2d ed., p. 626.)

Photographs of Jupiter made with the four ray-filters showed differences almost as marked as those found in the case of Saturn, though not so striking by casual inspection. The rotation of the planet is so rapid that it was found necessary to make the series of pictures as quickly as possible, if comparable results were to be obtained. Following was accomplished by bringing one of the outer satellites on the cross-hair of the eyepiece in the movable plate-holder. The exposure times were two seconds for the violet and yellow negatives, seven seconds for the ultra-violet, and ninety seconds for the infra-red. The dark belts are scarcely visible on the infra-red plates, while the pictures made with violet light show them in the greatest contrast.

This circumstance made it appear probable that very marked color contrasts would appear if the planet were viewed through a screen which transmitted deep-red and violet light. The violet filter used in the work has this property, and through it the cloud belts appeared deep red on a violet background. Unfortunately the vivid color contrast obtained in this way is not of much help in the visual study of the planet, as the eye does not focus well for these two regions of the spectrum.

The appearance of the disk in the light of the four spectral regions specified is shown on Plate V. They were all made on the evening of October 24, 1915, between 7:30 and 9:30 Pacific time. The details are given in Table I.

The direction of rotation of the planet is from right to left. On three of the plates slight blemishes have been noticed which have nothing to do with the surface markings: a light spot on the broad dark belt near the left hand edge, Fig. 8; a small dark spot near the lower edge, Fig. 11; and a small dark spot on the broad dark belt above the bright equatorial belt.

The chief differences between the views of the planet made by light of different wave-lengths may be summarized as follows: The infra-red pictures show less darkening of the disk as we approach

the limb, which is what would be expected on the theory that the darkening results from atmospheric absorption or scattering. The cloud belts moreover are very inconspicuous in these pictures. The two dark belts above the bright equatorial belt have a number of still darker spots on them which are most distinct in the yellow picture, less so in the violet, and almost invisible in the ultra-violet (Figs. 10, 11, 12). Above this dark belt, a double bright belt appears in the violet and ultra-violet pictures, which cannot be seen

TABLE I

Fig.	Ray-Filter	Time	Exposure	Plate
5	Infra-red	7:30	90 sec.	Spect. Panchrom.
6	Yellow	7:33	2	Cramer Iso. Inst.
7	Violet	8:15	2	Process
8	Ultra-violet	7:50	7	Process
9	Infra-red	9:20	90	Spect. Panchrom.
10	Yellow	9:23	2	Cramer Iso.
11	Violet	9:25	2	Process
12	Ultra-violet	9:30	7	Process

in the infra-red and yellow pictures, or, to speak more accurately, the dark belt between them is visible only in the violet and ultra-violet. The upper dark polar cap shades off gradually in the yellow and infra-red views, but is sharply terminated in the violet and ultra-violet, its lower edge marking the upper boundary of the double bright belt. These are the chief peculiarities. Observers who have made a long study of the surface markings of Jupiter will doubtless find other points of interest, and I shall be glad to lend the original plates to anyone who wishes to study them in greater detail.

TRICHROMATIC PHOTOGRAPHS OF THE PLANETS

By applying the methods of trichromatic color-photography to the negatives made through the ray-filters, very beautiful and striking transparencies have been made, which represent, in strong color contrasts, the differences which can be seen by comparison of the monochromatic photographs. As is well known, if three negatives are made of an object through red, green, and blue ray-filters, and the positives made from them on bichromatized gelatine are stained with blue, magenta, and yellow dyes, these positives

when superposed in register yield a fairly faithful reproduction of the object in its natural colors.

If we apply this same method to pictures made by yellow, violet, and ultra-violet light, we shall obtain a colored picture of the object as it would appear if the region of the eye's sensitiveness were shifted slightly toward the ultra-violet, or, to put the matter in astrophysical language, as the object would appear if it were receding from us with a velocity sufficient to cause ultra-violet to appear violet! Color-photographs of Saturn made in this way show the ball of the planet yellowish in color with a broad orange-red band (the violet and ultra-violet dark belt) surrounding the equator and a dark-red polar cap. The narrow belts, which can be seen visually and which appear on the negative made with yellow light, come out blue in the picture, the lower one being in coincidence with the upper edge of the dark belt. Owing to the fact that the dark belt is wider on the violet negative than on the ultra-violet one, the upper edge lacks yellow in the finished picture, and the superposed narrow blue belt appears purple in consequence (magenta+blue).

Color-contrasts also appear in the ring, as a result of the circumstance that the outer or fainter ring is relatively more luminous in ultra-violet light. The inner ring appears white, and the outer bluish green. Similar colored belts are shown in the pictures made of Jupiter. The photographs as a whole give us a vivid impression that we are really dealing with great belts of selectively absorbing, and hence colored, vapors or gases. They show us moreover that the absence of strong color-contrasts in the surface markings of the planets results merely from the limited range of the spectrum to which the eye is sensitive.

In view of the interesting results obtained with these two planets, it is my hope that similar observations will be made on Mars, on the occasion of its next near approach to the earth. I made one photograph of the planet with ultra-violet light in October, with the 60-inch reflector, but the disk was too small to show much of interest. Mr. Hoge was of very great assistance in helping me with the manipulation of the telescope.

JOHNS HOPKINS UNIVERSITY
February 1916

ORBIT OF THE SPECTROSCOPIC BINARY B.D. 78°412

By OLIVER J. LEE

The variable radial velocity of this star was announced in the *Astronomische Nachrichten*, **194**, 415, 1913. Its position is $a = 12^h8^m$; $\delta = 78°10'$. In the *Harvard Revision* its visual magnitude is given as 5.12 and its type as A 5. The spectrum contains numerous good lines, and on fully exposed plates these give reliable velocities. Several causes account for the fact that most of the plates in this series are slightly or appreciably underexposed. The photographic faintness of the star, 5.3 magnitude, the rapid changes in radial velocity and the desire to record these changes in as short times of exposure as possible, the northerly position of the star, keeping it always well above the horizon, with the accompanying tendency to follow the star into lower altitudes than would usually be the case—all of these contribute to the frequent large plate residuals shown in the last column of the summary of observations (Table II). The 60 plates discussed below were all taken with the one-prism arrangement of the Bruce spectrograph. The lines given in Table I were used to obtain the velocities.

TABLE I

Element	Wave-Length	Element	Wave-Length	Element	Wave-Length
Fe.....	4045.975	Ti.....	4395.201	Fe.....	4515.508
Fe.....	4063.759	Fe.....	4404.927	Ti+.....	4549.767
Fe.....	4071.908	Fe.....	4415.293	Ti.....	4572.156
Sr.....	4077.885	Mg.....	4481.400	Fe.....	4584.018
H.....	4101.900	Ti.....	4501.445	H.....	4861.527
H.....	4340.634	Fe.....	4508.455		

The hydrogen lines were weighted considerably lower than the others, for which the weights will average nearly equal. In the column headed "Observer," B = Barrett; F = Frost; L = Lee; M = S. A. Mitchell; My = C. A. Maney. Mr. F. R. Sullivan assisted as usual in securing the plates.

TABLE II

Summary of Observations

Plate No.	Observer	G.M.T.	Phase	No. of Lines	Wt.	Velocity	Residual O.−C.
		1913				Km	Km
IB 3252	L	Jan. 21.837	0.632	12	16	−55.6	+ 8.4
3270	L, M	Feb. 3.802	0.887	11	18	−13.9	+ 6.0
3279	B, L	5.800	0.343	12	14	+ 1.0	+ 8.4
3318	L	Mar. 17.667	0.809	6	7	−31.3	+ 9.6
3329	L	31.605	0.766	10	14	−55.2	− 5.7
3337	L	April 7.672	0.207	12	15	+39.1	+ 5.6
3341	L	15.638	0.547	11	13	−72.6	−15.6
3350	L	16.890	0.528	11	15	−61.0	− 6.5
3352	L	17.826	0.193	9	14	+46.4	+ 9.3
3356	B	18.854	1.221	17	21	+69.9	+ 7.9
3358	L	21.576	0.130	7	6	+39.0	−11.7
3363	B	25.710	0.451	10	14	−42.4	− 3.8
3366	L	28.577	0.776	11	10	−61.8	−13.4
3370	L, B	28.813	1.012	13	16	+13.7	− 4.8
		1914					
3644	L	Feb. 4.693	0.730	14	20	−64.3	− 7.6
3646	L, F	4.815	0.852	9	15	−27.7	+ 2.0
3649	F	4.994	1.031	13	21	+23.0	− 0.9
3652	L	13.522	0.662	12	21	−64.7	− 1.8
3654	L	13.650	0.790	12	17	−64.8	−19.4
3656	L	13.775	0.915	13	12	− 9.9	+ 2.1
3659	B	18.808	0.864	8	9	−17.3	+ 9.1
3660	B	18.984	1.040	15	20	+26.2	− 0.2
3664	L	20.660	0.174	12	18	+47.1	+ 5.6
3665	L	20.696	0.210	14	17	+23.7	− 9.0
3666	L	20.731	0.245	13	16	+21.7	− 1.3
3667	L	20.768	0.282	14	18	+ 6.8	− 3.8
3670	F	25.590	0.020	17	23	+70.0	+ 6.8
3671	F	25.635	0.065	14	17	+56.9	− 3.3
3672	F	25.681	0.111	12	16	+45.7	− 8.5
3674	B	25.831	0.261	13	18	+18.2	+ 0.4
3675	B	25.888	0.318	13	21	+ 4.3	+ 3.6
3677	L, F	27.771	0.930	11	14	− 9.4	− 1.8
3709	L	Mar. 20.917	0.469	5	5	−41.8	+ 0.7
3793	F	Aug. 3.841	0.396	4	5	− 7.3	+15.7
3794	F	3.889	0.444	3	4	−16.1	−20.6
3807	F	14.603	0.990	6	6	+ 9.6	− 2.2
3808	F	14.634	1.021	8	6	+16.5	− 4.9
3809	F	14.672	1.059	7	5	+25.3	− 6.4
3810	F	14.719	1.106	7	7	+43.1	− 0.5
		1915					
4116	My	April 16.776	0.860	7	10	−27.7	0.0
4117	B	16.820	0.904	9	12	− 9.9	+ 5.0
4118	B	16.868	0.952	8	13	+ 1.5	+ 2.2
4119	B	16.919	1.003	8	7	+ 5.0	−10.3
4128	My	27.580	0.225	8	14	+31.1	+ 2.4
4129	My	27.650	0.295	6	7	+ 0.7	− 6.6
4130	My	27.711	0.356	9	12	−16.2	− 4.8
4131	My	27.767	0.412	7	9	−27.0	+ 0.4
4132	B	27.822	0.467	8	10	−38.2	+ 3.8
4159	B	31.605	1.204	7	10	+68.4	+ 7.4

TABLE II—*Continued*

Plate No.	Observer	G.M.T.	Phase	No. of Lines	Wt.	Velocity	Residual O.−C.
		1913				Km	Km
4160.....	B	31.654	1.253	8	12	+65.0	+ 1.6
4161.....	B	31.707	0.035	10	16	+62.0	+ 0.8
4162.....	B	31.760	0.088	7	9	+54.3	− 3.4
4165.....	My	June 7.750	0.723	5	7	−41.6	+15.8
4166.....	My	7.803	0.776	6	10	−36.8	+11.6
4167.....	My	7.857	0.830	7	9	−32.2	+ 3.4
4185.....	My	July 5.621	0.632	9	11	−67.9	− 4.0
4186.....	My	5.677	0.688	7	8	−50.4	+10.6
4187.....	My	5.731	0.742	9	11	−57.6	− 2.9
4188.....	My	5.787	0.798	7	12	−39.0	+ 4.6
4189.....	My	5.842	0.853	7	9	−25.1	+ 4.4

The observations cover a time of 895 days corresponding to 705 periods. The period of 1.27100 days is considered definitive, and its error can hardly exceed ±2 units in the last place. With this period the observations were plotted and grouped into normal

TABLE III

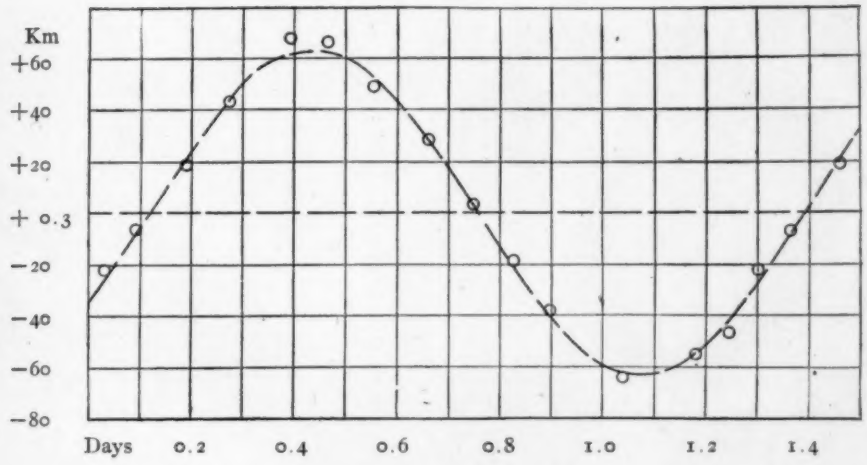
NORMAL PLACES

No.	Phase	Limits of Phase	Velocity	Residual O.−C.	Wt.
	Days		Km	Km	
1.....	0.027	$1^d.25$–$0^d.06$	+66.8	+3.8	0.4
2.....	0.114	0.06–0.19	+49.6	−4.1	.7
3.....	0.223	0.19–0.27	+29.3	+0.5	.9
4.....	0.309	0.27–0.35	+ 3.8	+0.7	.6
5.....	0.388	0.35–0.43	−18.2	+3.0	.3
6.....	0.458	0.43–0.52	−37.8	+2.3	.3
7.....	0.600	0.52–0.69	−63.9	−2.0	.8
8.....	0.743	0.69–0.78	−54.7	−0.5	.8
9.....	0.807	0.78–0.83	−46.2	−4.7	.4
10.....	0.863	0.83–0.89	−21.7	+5.3	.6
11.....	0.925	0.89–0.96	− 6.8	+1.8	.5
12.....	1.022	0.96–1.07	+19.1	−2.1	.8
13.....	1.106	1.10	+43.1	−0.5	.1
14.....	1.226	1.18–1.25	+68.2	+6.3	0.4

points with regard to phase, after the times of observation had been corrected for the light-equation. The smooth curve drawn to represent these points promised an appreciable eccentricity, and elliptical elements were derived by use of the graphical method of Lehmann Filhés. It was seen that a sine curve suited the observa-

tions even better than the curve from these elements. An ephemeris for the normal points was computed from the following elements of a circular orbit and the differences from the observed values have been adjusted by a least-squares solution:

$$P = 1.27100 \text{ days}$$
$$K = 65.5 \text{ km}$$
$$\gamma = +1.5 \text{ km}$$
$$T = \text{J.D. } 2420685.266$$



Velocity-Curve of B.D. 78°412

The normal equations

$$\begin{aligned} 7.6\,x - 0.225\,y - 0.173\,z &= -9.02 \\ 3.439\,\; + 0.838\,\; &= -8.01 \\ 4.174\,\; &= -3.05 \end{aligned}$$

in which $x = \delta\gamma$, $y = \delta K$, $z = K\mu\delta T$ gives the following values of the corrections:

$$\delta\gamma = -1.2 \text{ km}$$
$$\delta K = -2.3 \text{ km}$$
$$\delta T = -0.001 \text{ day}$$

The final elements are:

$$P = 1.27100 \text{ days} \pm 0.00002 \text{ day}$$
$$\mu = 283^\circ.24$$
$$e = 0.00$$
$$K = 63.2 \text{ km} \pm 1.44 \text{ km}$$
$$\gamma = +0.3 \text{ km} \pm 0.94 \text{ km}$$
$$T = \text{J.D. } 2420685.265 \pm 0.0005$$
$$a \sin i = 1{,}104{,}000 \text{ km}$$

T is taken as a time of maximum velocity of recession.

The phases given in column 4 of Table II are referred to T as defined above. The quantities O.—C. in the last column of the same table are scaled off with the velocity-curve from the final elements. From them the probable error of a plate is ± 5.2 km. Although this quantity is large and the probable error of a normal point is ± 1.9 km, the orbit may be considered definitive.

Mr. Julius Lemkowitz, computer at this observatory, has kindly checked the least-squares solution.

YERKES OBSERVATORY
May 4, 1916